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Math170: Mathematical Methods for Optimization Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Your Name: _____

Your SID: _____

1. Let $A \in \mathbf{R}^{m \times n}$, and $l, u \in \mathbf{R}^n$ with $m < n$. Find the dual of the linear program

$$\min c^T x$$
$$\text{s.t.} \quad Ax = b,$$
$$l \le x \le u.$$

2. Let $A \in \mathcal{R}^{n \times m}$, $B \in \mathcal{R}^{k \times m}$ and $b \in \mathcal{R}^n$. Show that

   - either there exists a vector x such that $Ax = b$ with $Bx \geq 0$,

   - or (exclusive) there exist vectors y and w with $w \geq 0$ such that $A^T y = B^T w$ and $b^T y < 0$.

   Hint: Introduce slack variables to put the first part in a form consistent with the Farkas Lemma.

3. Let
$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 4 \\ 7 \end{pmatrix}.$$

Consider the linear program

$$\max c^T x$$
$$\text{s.t.} \quad Ax = b,$$
$$x \geq 0.$$

Find out conditions on c under which the basic feasible solution $x^0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is the optimal solution.

4. Solve the following linear program

$$\min c^T x$$
$$s.t. \quad A \geq b$$
$$x \geq 0,$$

where

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 1 \\ 1 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$