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Math170: Mathematical Methods for Optimization
Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	24	
3	24	
4	24	
5	24	
Total	100	

1.
Your Name: ______________________________
Your SID: ______________________________

2. Given the basic feasible solution $x^T = (1, 0, 2)$. Solve the linear program

$$\begin{aligned}&\min x_1 + x_2 + x_3,\\ &s.t. \quad x_1 + x_2 = 1, \quad x_2 + x_3 = 2,\\ &x_1, x_2, x_3 \geq 0.\end{aligned}$$

3. Given a linear program

$$
\begin{aligned}
&\min c^T x \\
&s.t. \quad A_1 x + B_1 z = b_1, \\
&A_2 x + B_2 z \leq b_2, \\
&x \geq 0.
\end{aligned}
$$

Rewrite this linear program in canonical form.

4. Suppose A is skew-symmetric, that is $A^T = -A$. Consider the linear program

$$\begin{aligned} &\min b^T x \\ &s.t. \quad Ax = b, x \geq 0. \end{aligned}$$

Show that any feasible x is in fact an optimal solution.

5. Let $A \in \mathcal{R}^{n\times m}$, $B \in \mathcal{R}^{n\times k}$ and $b \in \mathcal{R}^n$. Show that the system

$$Ax + By = b \quad \text{has no solution for } x \geq 0, \text{ where } x \in \mathcal{R}^m, y \in \mathcal{R}^k,$$

if and only if the system

$$A^T z \geq 0, \quad B^T z = 0 \quad \text{and} \quad b^T z < 0$$

has a solution $z \in \mathcal{R}^n$.