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Math170: Mathematical Methods for Optimization Final

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	2	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
8	14	
Total	100	

1. Your Name: ______________________________

   Your SID: ______________________________

2. Let A be a skew-symmetric matrix: $A = -A^T$. Consider the linear program

$$\begin{aligned} &\max b^T x, \\ &s.t. \quad Ax \geq b, \\ &x \geq 0. \end{aligned}$$

Show that the dual of this linear program can be expressed in exactly the same form.

3. You are the manager of a large company where you face the decision of selecting the right projects to maximize the total returns. There are n possible projects P_k, $k = 1, \cdots, n$. Each project P_k runs for 3 years and has an overall return of c_k dollars. The financial constraints are that in year t there are only a total of f_t dollars available for these projects, whereas project P_k requires at least $\alpha_{k,t}$ dollars (for $t = 1, 2, 3$ and $k = 1, \cdots, n$.) Formulate this problem as an integer program. **Hint:** Define variables x_k so that $x_k = 1$ means selecting P_k and $x_k = 0$ means not selecting P_k. Formulate your integer program in terms of these variables.

4. Let S be any non-empty set in $\mathbf{R}^n$. Let C consist of all convex combinations

$$\theta_1 x_1 + \cdots + \theta_k x_k, \quad \text{with} \quad \theta_i \geq 0, \quad \sum_i \theta_i = 1, x_i \in S.$$

   Show that C is convex.

5. Draw the feasible solutions x

$$3x_1 + x_2 \geq 6, \quad x_1 \geq 0, x_2 \geq 0.$$

Suppose the composite cost is

$$Cx = \begin{pmatrix} 5x_1 + x_2 \\ x_1 + 2x_2 \end{pmatrix}.$$

Find all the efficient points x.

6.
- State the zero-sum, two-person game.
- State the zero-sum, two-person symmetric game.
- Show that the value of the zero-sum, two-person symmetric game is zero.

7. The transportation problem in canonical form takes the form

$$
\begin{aligned}
\min \sum_{i,j} & c_{i,j} x_{i,j}, \\
s.t. \quad & \sum_j x_{i,j} = s_i, \quad i = 1, \cdots, m, \\
& \sum_i x_{i,j} = d_j, \quad j = 1, \cdots, n, \\
& x_{i,j} \geq 0.
\end{aligned}
$$

- Under what conditions is this problem feasible?
- Assume the problem is feasible. Show that its constraints are redundant.

8. • For any vector $u = (u_1, \cdots, u_n)^T$, define $\|u\|_1 = |u_1| + \cdots |u_n|$. Consider the problem

$$\begin{aligned} &\min_x \|Mx - g\|_1 \\ &s.t. \quad Ax = b, \quad x \geq 0. \end{aligned}$$

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Rewrite this problem as a linear program.

• Consider constrainted least squares problem

$$\begin{aligned} &\min_x \|Mx - g\|^2 \\ &s.t. \quad Ax = b, \quad x \geq 0. \end{aligned}$$

Rewrite this problem as a quadratic program. Find necessary conditions for optimality.