Homework 9 Math 170
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Problem set: *Franklin* Section 1.16, Problems 1,2,3,7,10

Problems checked: *Franklin* Section 1.16, Problems 1,2

Grading scheme:

X	for "complete": significant effort demonstrated
O	for "fail": lack of demonstration of significant effort

Problems graded: *Franklin* Section 1.16, Problems 3,7,10

Grading scheme:

3	for "excellent":	Necessary steps are all shown and well explained. Solution is correct.
2	for "fair":	Necessary steps are all shown. There are minor gaps in explanantion and/or minor errors in solution.
1	for "poor":	Necessary steps are lacking. There are major gaps in explanantion and/or major errors in solution.
0	for "fail":	Significant effort is not demonstrated.

Sample solutions:

***Franklin* Section 1.16, Problem 3** Start with the zero flow. There is an unsaturated path $s\,a\,s'$, which gives the new flow with $f(s,a)-f(s,a)=f(a,s')=-f(s',a)=2$ and all other $f(x,y)=0$. There is still an unsaturated path $s\,a\,b\,s'$, which gives a new flow with

$$f(s,a)=-f(a,s)=3,\ f(a,s')=-f(s',a)=2,\ f(a,b)=-f(b,a)=1,$$
$$f(b,s')=-f(s',b)=1$$

and all other $f(x,y)=0$. There is still an unsaturated path $s\,b\,s'$, which gives rise to a new flow with

$$f(s,a)=-f(a,s)=3,\ f(s,b)=-f(b,s)=7,\ f(a,s')=-f(s',a)=2,$$
$$f(a,b)=-f(b,a)=1,\ f(b,s')=-f(s',b)=8$$

and all other $f(x,y)=0$. There are no remaining unsaturated paths, so we are done and this is an optimal flow with value $3+7=10$.

***Franklin* Section 1.16, Problem 7** Number the nodes $\{s,a,b,s'\}\to\{1,2,3,4\}$. Then we have the linear program (leaving out the variables which are necessarily zero, $\{x_{14},x_{21},x_{31},x_{41},x_{42},x_{43}\}$),

$$\begin{bmatrix} x_{12}\\ x_{13}\\ x_{23}\\ x_{24}\\ x_{32}\\ x_{34}\end{bmatrix}\geq 0,\ \begin{bmatrix} x_{12}\\ x_{13}\\ x_{23}\\ x_{24}\\ x_{32}\\ x_{34}\end{bmatrix}\leq\begin{bmatrix}5\\1\\2\\1\\4\\4\end{bmatrix},\begin{bmatrix}-1&0&1&1&-1&0\\0&-1&-1&0&1&1\end{bmatrix}\begin{bmatrix} x_{12}\\ x_{13}\\ x_{23}\\ x_{24}\\ x_{32}\\ x_{34}\end{bmatrix}=0,$$

$$\text{maximize } (x_{12}+x_{13})$$

***Franklin* Section 1.16, Problem 10** Add a new source with capacity to each of the current sources equal to the sum of capacities out of that source and zero return capacity and convert all of the current sources into regular nodes. Similarly, add a new sink with capacity from each of the current sinks equal to the sum of capacities into that sink and zero capacity back and convert all the current sinks into regular nodes.