Homework 10 Math 170
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Problem set: *Franklin* Section 1.17, Problems 1,2,3,4,5,6,9,11

Problems checked: *Franklin* Section 1.17, Problems 1,2,3,5,6

Grading scheme:

X	for "complete": significant effort demonstrated
O	for "fail": lack of demonstration of significant effort

Problems graded: *Franklin* Section 1.17, Problems 4,9,11

Grading scheme:

3	for "excellent":	Necessary steps are all shown and well explained. Solution is correct.
2	for "fair":	Necessary steps are all shown. There are minor gaps in explanantion and/or minor errors in solution.
1	for "poor":	Necessary steps are lacking. There are major gaps in explanantion and/or major errors in solution.
0	for "fail":	Significant effort is not demonstrated.

Sample solutions:

***Franklin* Section 1.17, Problem 4** Following example 2 in the text, start with

$$\begin{matrix} & 0 & 0 & 0 \\ 3 & 3 & 3 & 2 \\ 5 & 1 & 3 & 5 \\ 3 & 2 & 1 & 3 \end{matrix}, \text{qualificationmatrix} \begin{matrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{matrix}$$

Complete assignment is impossible, so we need to continue. It is possible to increase v_3 and decrease u_2 and u_3 by one without violating the constraint, giving

$$\begin{matrix} & 0 & 0 & 1 \\ 3 & 3 & 3 & 2 \\ 4 & 1 & 3 & 5 \\ 2 & 2 & 1 & 3 \end{matrix}, \text{qualificationmatrix} \begin{matrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{matrix}$$

Complete assigment is now possible, with the 2 in the first column, the 3 in the top spot of the second column and the 5 in the third column, giving a maximum sum of 10 for the OK subsets. This solution is readily apparent by inspection.

***Franklin* Section 1.17, Problem 9** Clearly, for any c_1, c_2 in the network, the shortest path from a to c_2 is less than or equal to the the shortest path from a to c_1 plus $\tau(c_1, c_2)$, so ϕ is an admissable potential. For any $\lambda \neq \omega$, $\phi(\omega) \leq \phi(\lambda) + \tau(\lambda, \omega)$, so this also holds for the minimizing λ. On the other hand, for λ the preceding node to ω on the minimizing path from a to ω, equality holds, so $\phi(\omega)$ is greater than or equal to the minimum over all $\lambda \neq \omega$. Hence,the functional equation holds.

***Franklin* Section 1.17, Problem 11** Letting $\otimes$ represent the Kronecker product of matrices (look this up if you are not familiar with it; it is a very handy thing to know), Vec be the operation that stacks the columns of a matrix to form a column vector, I_n be the $n \times n$ identity matrix, and e_n be the row matrix of n ones, then the transportation problem can be written in block form as the canonical optimization problem

$$\begin{bmatrix} e_m \otimes I_n \\ I_m \otimes e_n \end{bmatrix} \mathrm{VecX} = \begin{bmatrix} s \\ d \end{bmatrix}, \mathrm{VecX} \geq 0, \mathrm{minimize}(\mathrm{VecC})^{\mathrm{T}} \mathrm{VecX}$$

with dual problem

$$\begin{bmatrix} u^{\mathrm{T}} & v^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} e_m \otimes I_n \\ I_m \otimes e_n \end{bmatrix} \leq (\mathrm{VecC})^{\mathrm{T}}, \mathrm{maximize} \begin{bmatrix} u^{\mathrm{T}} & v^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} s \\ d \end{bmatrix}$$

which written in indicial notation is what is asked to demonstrate.