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Math128A: Numerical Analysis Sample Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	16	
Total	100	

Your Name: ______________________________

Your SID: ______________________________

Your GSI: ______________________________

1. Show that the cubic equation $2x^3 - 6x + 1 = 0$ has a real root in the interval $[0 \quad 1/2]$. Perform one step of Bisection method with this interval.

2. Let $x_0 < x_1 < x_2$. Find a second degree polynomial $P(x)$ such that

$$P(x_0) = f_0, \quad P(x_1) = f_1, \quad \text{and} \quad P'(x_2) = f_2'.$$

Hint: Write $P(x)$ as

$$P(x) = \alpha + \beta(x - x_0) + \gamma(x - x_0)(x - x_1)$$

and then determine the coefficients from the given conditions.

3. Define the absolute error, the relative error, and the number of significant digits.

4. Boole's Rule for numerical integration on the interval $[a, b]$ is given by

$$I_4(x) = \frac{2h}{45}\left(7f(a) + 32f(a+h) + 12f(a+2h) + 32f(a+3h) + 7f(b)\right).$$

   (a) Show that the degree of precision of this formula is 5.

   (b) Develop the Composite Boole's Rule for integration on $[a, b]$.

5. Suppose that

$$L = \lim_{h \to 0} f(h) \quad \text{and} \quad L - f(h) = c_6 h^6 + c_9 h^9 + \cdots .$$

Find a combination of $f(h)$ and $f(h/2)$ that is a much better estimate of L.

6. Given $h = 0.1$, $f(0) = 0$, $f(0.1) = 0.01$ and $f(0.2) = 0.04$, find a second order approximation to $f'(0)$.

7. Find the Lipschitz constant for the given function $f(t,y) = t\mathbf{sin}(y^2)$ on domain

$$D = \{(t,y)\,|0 \leq t \leq 1, -1 < y < 1\}.$$

Show that $f(t,y)$ does not satisfy the Lipschitz condition on

$$D = \{(t,y)\,|0 \leq t \leq 1, -\infty < y < \infty\}.$$