Prof. Ming Gu, 861 Evans, tel: 2-3145
Email: mgu@math.berkeley.edu
http://www.math.berkeley.edu/∼mgu/MA128ASpring2017

Math128A: Numerical Analysis Sample Final

This is a closed book, closed notes exam, with the exception of a one-sided one-page cheat sheet. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Your Name: _____

Your SID: _____

1. (a) Describe a method to numerically evaluate $\int_1^\infty \frac{e^{-x}}{x^2}$ with composite Simpson's Method. No actual calculation is required.

    (b) Evaluate

$$\int_{-1}^1 \int_{-2}^2 \left(x^2 + y^2 + xy \right) dx dy.$$

2. Given three distinct points x_0, x_1, and x_2, develop a finite difference scheme to approximate $f''(x_0)$.

   (a) What is the order of your method?

   (b) Discuss conditions under which your approximation is second order.

3. Let

$$A = \begin{pmatrix} d_1 & \alpha_2 & \cdots & \alpha_{n-1} & \alpha_n \\ \beta_2 & d_2 & & & \\ \vdots & & \ddots & & \\ \beta_{n-1} & & & d_{n-1} & \\ \beta_n & & & & d_{n-1} \end{pmatrix} \in \mathbf{R}^{n \times n}$$

be an *arrow matrix*. Develop an algorithm to LU-factorize A in $O(n)$ operations.

4. (a) For a function f and distinct points α, β, and γ, define what is meant by $f[\alpha, \beta, \gamma]$.

   (b) Find the Lagrange form of the polynomial $P(x)$ which interpolates $f(x) = 4x/(x+1)$ at $0, 1,$ and 3.

5. (a) Define what is meant by the local truncation error, and the order, for a single-step method for solving the ODE's.

   (b) Derive a specific Runge-Kutta method of local order 2. Show your work.

6. Consider the iteration

$$x_{k+1} = \alpha x_k + \beta, \quad k = 0, 1, \cdots,$$

with $|\alpha| < 1$.

(a) Assume that the iteration converges, what is the limit?

(b) What is the order of convergence of this iteration?