Prof. Ming Gu, 861 Evans, tel: 2-3145
Email: mgu@math.berkeley.edu
http://www.math.berkeley.edu/~mgu/MA128A2012S

Math128A: Numerical Analysis Sample Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Your Name: ____________________________

Your SID: ____________________________

Your GSI: ____________________________

1. (a) Show that the cubic equation $2x^3 - 6x + 1 = 0$ has a real root in the interval $[0 \quad 1/2]$. Perform one step of Bisection method with this interval.

   (b) Reformulate the above equation as

$$x = \frac{2x^3 + 1}{6}.$$

   Define the fixed point iteration (FPI) based on this equation, and show that FPI convergences for any initial guess in $[0 \quad 1/2]$.

2. Let $x_0 < x_1 < x_2$. Find a second degree polynomial $P(x)$ such that

$$P(x_0) = f_0, \quad P(x_1) = f_1, \quad \text{and} \quad P'(x_2) = f_2'.$$

Hint: Write $P(x)$ as

$$P(x) = \alpha + \beta(x - x_0) + \gamma(x - x_0)(x - x_1)$$

and then determine the coefficients from the given conditions.

3. Define the absolute error, the relative error, and the number of significant digits.

4. Determine the value of n and h required to approximate

$$\int_0^2 \frac{1}{x+4}dx$$

to within 10^{-5} using Composite Simpson's rule.