

Prof. Ming Gu, 861 Evans, tel: 2-3145  
Email: mgu@math.berkeley.edu  
<http://www.math.berkeley.edu/~mgu/MA128A2008S>

## Math128A: Numerical Analysis Sample Midterm I

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This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	24	
3	24	
4	24	
5	24	
Total	100	

1. (4 Points)

Your Name: \_\_\_\_\_

Your SID: \_\_\_\_\_

Your GSI: \_\_\_\_\_

2. (24 Points) Show that the cubic equation  $2x^3 - 6x + 1 = 0$  has a real root in the interval  $[0, 1]$ . Perform one step of Bisection method with this interval. Reformulate this equation as a fixed point equation, and perform one step of fixed point iteration.

3. (24 Points) Let  $x_0 < x_1 < x_2$ . Find a second degree polynomial  $P(x)$  such that

$$P(x_0) = f_0, \quad P(x_1) = f_1, \quad \text{and} \quad P'(x_2) = f'_2.$$

**Hint:** Write  $P(x)$  as

$$P(x) = \alpha + \beta(x - x_0) + \gamma(x - x_0)(x - x_1)$$

and then determine the coefficients from the given conditions.

4. (24 Points) Let  $a > 0$ . The cubic root  $a^{1/3}$  is the unique positive root of the equation

$$x^3 - a = 0.$$

- (a) Define the Newton iteration for solving this equation.
- (b) Given  $a = 2$  and  $x_0 = 1$ , compute the first two iterates  $x_1$  and  $x_2$  in the Newton iteration.
- (c) Define the order of convergence for any convergent iteration. Show this iteration is 2nd order convergent.

5. (24 Points) Define the absolute error, the relative error, and the number of significant digits.