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Math128A: Numerical Analysis
Programming Assignment #3, Due May 7, 2008

In classical mechanics, the two-body problem is to determine the motion of two point particles that interact only with each other. An example is the moon orbiting the earth.

Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the positions of the two bodies, and m_1 and m_2 be their masses. We would like to determine the trajectories $x_1(t)$ and $x_2(t)$ for time t, given initial positions $\mathbf{x}_1(t == 0)$, $\mathbf{x}_2(t == 0)$, and initial velocities $\mathbf{v}_1(t == 0)$, $\mathbf{v}_2(t == 0)$. According to Newton's second law,

$$F_{12}(\mathbf{x}_1, \mathbf{x}_2) = m_1 \ddot{\mathbf{x}}_1,$$
$$F_{21}(\mathbf{x}_1, \mathbf{x}_2) = m_2 \ddot{\mathbf{x}}_2,$$

where $F_{12}(\mathbf{x}_1, \mathbf{x}_2)$ is the force on mass 1 due to its interaction with mass 2; and $F_{21}(\mathbf{x}_1, \mathbf{x}_2) = -F_{12}(\mathbf{x}_1, \mathbf{x}_2)$ is the force on mass 2 due to its interaction with mass 1.

For this assignment, we assume that

- The two masses are on a plane, so that $\mathbf{x}_1$ and $\mathbf{x}_2$ have two components each.
- $$F_{12}(\mathbf{x}_1, \mathbf{x}_2) = \frac{\gamma m_1 m_2}{r^\alpha} \left(\mathbf{x}_2 - \mathbf{x}_1 \right),$$

  where α is a given constant and $r = \|\mathbf{x}_2 - \mathbf{x}_1\|$.

We will assume that $m_1 = 50$, $m_2 = 100$, $\gamma = 10^2$, and

$$\mathbf{x}_1(t == 0) = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad \mathbf{x}_2(t == 0) = \begin{pmatrix} -10 \\ 0 \end{pmatrix}, \quad \mathbf{v}_1(t == 0) = \begin{pmatrix} 0 \\ 10 \end{pmatrix}, \quad \mathbf{v}_2(t == 0) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}.$$

- Reformulate this problem as a system of first order ODEs.
- For $\alpha = 2, 3, 4$, solve the system of ODEs for 30 orbits using Algorithm 5.7 in the text (with $h \leq 10^{-2}$) and `ode45` in matlab. Describe the motion of these masses. You should see a collision for $\alpha = 2$, an elliptical orbit for $\alpha = 3$, and the particles moving apart for $\alpha = 4$.