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Math128A: Numerical Analysis Midterm I

   This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	24	
3	24	
4	24	
5	24	
Total	100	

1. (4 Points)

   Your Name: _____

   Your SID: _____

   Your GSI: _____

2. (24 Points) Consider the fixed point iteration

$$p_{k+1} = (3p_k + 1)^{\frac{1}{3}}, \quad k = 0, 1, \cdots,$$

(a) Show that there is a fixed point in $p_0 \in [1, 2]$.

(b) Show that the above iteration converges for any $p_0 \in [1, 2]$.

3. (24 Points) Find a polynomial $P(x)$ of degree at most 2 such that

$$P(0) = 1, \quad P(1) = 1, \quad \text{and} \quad P(2) = 1.$$

4. (24 Points) Find the largest interval such that for all p^* in this interval p^* approximates $p = \sqrt{2}$ with relative error at most 10^{-4}. You do not need to simplify your expressions for the interval. Show that any p^* in this interval will have at least 4 significant digits.

5. (24 Points) Let $\alpha > 1$ and $\lambda > 0$. Consider the following sequence

$$p_k = \lambda^{\alpha^k}, \quad k = 1, 2, \cdots, .$$

(a) Find conditions under which
$$\lim_{k \to \infty} p_k = 0.$$

(b) Assume that $\lim_{k \to \infty} p_k = 0$, find the order of convergence of this sequence.