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Math128A: Numerical Analysis Sample Midterm II

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	24	
3	24	
4	24	
5	24	
Total	100	

1. (10 Points)

   Your Name: _____

   Your SID: _____

   Your GSI: _____

2. Boole's Rule for numerical integration on the interval $[a, b]$ is given by

$$I_4(x) = \frac{2h}{45}\left(7f(a) + 32f(a+h) + 12f(a+2h) + 32f(a+3h) + 7f(b)\right).$$

   (a) Show that the degree of precision of this formula is 5.

   (b) Develop the Composite Boole's Rule for integration on $[a, b]$.

3. Construct the natural cubic spline that approximates

$$f(x) = \frac{\sin x}{x}$$

at the nodes $-1, 0, 1$.

4. Suppose that
$$L = \lim_{h \to 0} f(h) \quad \text{and} \quad L - f(h) = c_6 h^6 + c_9 h^9 + \cdots.$$
Find a combination of $f(h)$ and $f(h/2)$ that is a much better estimate of L.

5. Consider the initial value problem $y'(t) = f(t, y) = t * y + 1$ for $t \in [0, 1]$ and $y(0) = 1$.
   (a) Show that $f(t, y)$ satisfies the Lipschitz condition for $0 \leq t \leq 1$ and $-\infty < y < \infty$.
   (b) For $h = 0.2$, find an approximation to $y(h)$ using the Euler's method.