Prof. Ming Gu, 861 Evans, tel: 2-3145
Email: mgu@math.berkeley.edu
http://www.math.berkeley.edu/∼mgu/MA128A

Math128A: Numerical Analysis Programming Assignment 4

The Lotka-Volterra equations describe a population dynamics model. The prey population (x) would grow naturally if the predator (y) were not around, and the predator population would gradually die out without sufficient supply of prey. On the other hand, the predator would successfully grab its meal for a certain fraction of the encounters between the prey and predator, decreasing prey population and increasing predator population. The equations are

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= -\gamma y + \delta xy, \end{aligned}$$

where α, β, γ and δ are positive constants.

1. Show that the prey and predator populations satisfy the following equation

   $$-\gamma \log x + \delta x = \alpha \log y - \beta y + C,$$

   for some constant C[1].

2. Let initial condition be $x_0 = 1$, $y_0 = 1$; let t be in between 0 and 4; and let $\beta = \delta = 1$. We solve the Lotka-Volterra equations for three cases with $\alpha = \gamma = \tau$ for $\tau = 10$, 100, and 1000. The following methods should be used:

   - Algorithm 5.2 on Page 278.
   - Algorithm 5.3 on Page 285.
   - Matlab functions `ode45`, `ode23s`, and `ode113`.

   Choose stepsize to ensure convergence. Use the methods in the text book with caution as they were developed for the single variable case. Report the number of function evaluations for each solution and each method and plot the solutions on the xy plane as well as the 3-dimensional xyt space.

---

[1] It is known that this equation in general defines a closed curve on the xy plane. The solutions to the Lotka-Volterra equations are periodic, indicating that the predator and prey populations undergo periodic increases and decreases.