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Math128A: Numerical Analysis Midterm I

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	24	
3	24	
4	24	
5	24	
Total	100	

1. (4 Points)

Your Name: _____

Your SID: _____

2. (24 Points) Find out the number of real roots of the cubic equation $2x^3 - 6x + 1 = 0$. Justify your answer.

3. (24 Points) Let $x_0 < x_1 < x_2$. Find a second degree polynomial $P(x)$ such that

$$P(x_0) = f_0, \quad P(x_1) = f_1, \quad \text{and} \quad P'(x_2) = f'_2.$$

Hint: Write $P(x)$ as

$$P(x) = \alpha + \beta(x - x_0) + \gamma(x - x_0)(x - x_1)$$

and then determine the coefficients from the given conditions.

4. (24 Points) Let $a > 0$. The cubic root $a^{1/3}$ is the unique positive root of the equation

$$x^3 - a = 0.$$

- (a) Define the Newton iteration for solving this equation.
- (b) Given $a = 2$ and $x_0 = 1$, compute the first two iterates x_1 and x_2 in the Newton iteration.
- (c) Define the order of convergence for any convergent iteration. Show this iteration is 2nd order convergent.

5. (24 Points) Let $0 < \alpha < 10^{-8}$ be any floating point number. Show that in double precision arithmetic, $\mathbf{fl}(\sin(\alpha)) = \alpha$.