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Math128A: Numerical Analysis Sample Final

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	4	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
Total	100	

1. (4 Points)

   Your Name: ______________________________

   Your SID: ______________________________

2. (12 Points)

   (a) Describe a method to evaluate $\int_1^{\infty} \frac{e^{-x}}{x^2}$. No actual calculation is required.

   (b) Evaluate

$$\int_{-1}^{1} \int_{-2}^{2} \left(x^2 + y^2 + xy\right) dxdy.$$

3. (12 Points)

   (a) From the Taylor expansion of a function $f(x)$, derive a first-order approximation to $f'(x)$.

   (b) Use Richardson's extrapolation method to find a 3 point 2nd order formula.

4. (12 Points)

   (a) Let A and B be $n \times n$ matrices. Prove or find a counter example: If $AB = 0$ then $A = 0$ or $B = 0$.

   (b) Let A and B be $n \times n$ matrices. Prove or find a counter example: If $AB = 0$ then $\det(A) = 0$ or $\det(B) = 0$.

5. Consider the iteration

$$x_{k+1} = 2x_k - \alpha x_k^2, \quad k = 0, 1, \cdots,$$

where $\alpha > 0$ is given. Show that the iteration converges quadratically to $1/\alpha$ for any initial guess x_0 satisfying $0 < x_0 < 2/\alpha$.

6. (12 Points)

   (a) For a function f and distinct points α, β, and γ, define what is meant by $f[\alpha, \beta, \gamma]$.

   (b) Find the Lagrange form of the polynomial $P(x)$ which interpolates $f(x) = 4x/(x+1)$ at $0, 1$, and 3.

7. For the following linear system

$$\begin{aligned} x - \alpha y &= 1, \\ \alpha x - y &= 1, \end{aligned}$$

describe for which values of α the system has an infinite number of solutions, no solutions, and exactly one solution, and find the solution when it is unique.

8. Determine the free cubic spline that apprixmates the data $f(-1) = 1$, $f(0) = 0$ and $f(1) = 1$.

9. (a) Define what is meant by the local truncation error, and the local order, for a single-step method for solving the ODE's.

   (b) Derive a specific Runge-Kutta method of local order 2. Show your work.