

## Syllabus for Qualifying Exam

Lynn Scow  
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Committee: Thomas Scanlon, Leo Harrington, George Bergman (Chair), John MacFarlane (Philosophy)

**Major Topic:** *Model Theory* (foundations)

reference: Marker, David. Model Theory: An Introduction. Ch.s 1-4 [not differentially closed fields]; 5.1-5.3; 6; 7.1-7.3, 7.5. [exercises excluded]

- (i) Compactness, Complete theories, Craig/Robinson/Beth, Upward/Downward Lowenheim Skolem Theorems, Model companions, Ehrenfeucht-Fraïssé games.
- (ii) Quantifier elimination, Algebraically closed fields, Real closed fields, Omitting Types Theorem, Prime and atomic models,  $\kappa$ -saturated/homogeneous/universal models,  $\omega$ -stability, Indiscernible sequences.
- (iii) The Categoricity Theorem, Morley rank, Definability of types, Basic facts about  $\omega$ -stable groups, Infinity-definable groups in  $\omega$ -stable structures are definable, Hrushovski's Theorem (Thm 7.5.4 in Marker).

**Advanced Topic:** *Ultraproducts and Saturation* (foundations)

reference: Shelah, Saharon. Classification Theory. Ch.s VI.1, VI.2 [not Theorem 2.6], VI.3 [up to the Keisler-Shelah Theorem in Exercise 3.5].

- (i)  $\lambda$ -regular filters and saturation properties of ultraproducts, Games  $G_L^\alpha(M, N)$  and compactness of ultraproducts,  $\lambda$ -good filters and saturation properties of ultraproducts
- (ii) Martin's Axiom and the saturation of ultraproducts,  $\kappa$ -independent families  $(F, G, D)$ , Keisler-Shelah Theorem.

**Minor Topic:** *General Algebra* (algebra)

reference: Bergman, George. An Invitation to General Algebra and Universal Constructions. Ch.s 6.5, 6.8, 6.9; 8.1-8.5, 8.6, 8.9. [without exercises]

- (i) Functors, Universal objects, Morphisms of functors, Equivalence of categories.
- (ii)  $\Omega$ -algebras, Terms, Varieties, Equational theories, Birkhoff's Theorem, Derived operations, Clones.