Homework 3, Math 277, due Friday, March 20.

Given $\kappa > 0$, an ancient solution is *κ-noncollapsed at all scales* if : For any $r > 0$ and any radius-r metric ball B in a time slice, for which $|Riem| \leq r^{-2}$ on B, we have $r^{-n}\,\mathrm{vol}(B) \geq \kappa$.

Any blowup limit of a finite-time singularity is an ancient solution which is κ-noncollapsed at all scales for some $\kappa > 0$ (provided that the original Ricci flow is on a compact manifold).

The following manifolds have standard ancient solutions. For which of them is there some $\kappa > 0$ so that it is κ-noncollapsed at all scales? Justify your answer.

1. $\mathbb{R}^n$.

2. $\mathbb{R}^{n-1} \times S^1$.

3. T^n.

4. S^n, where $n \geq 2$.

5. $\mathbb{R} \times S^{n-1}$, where $n \geq 3$.

6. The cigar soliton.

7. $\mathbb{R} \times (\textit{cigar soliton})$.

8. The Bryant soliton. (This is a 3-dimensional rotationally symmetric gradient steady soliton. The metric at time 0 takes the form $g = dr^2 + \mu(r) h_{S^2}$, where h_{S^2} is the standard metric on S^2 and $\mu(r) \sim r$ for large r. Its sectional curvatures decay like r^{-1}.)