h-PRINCIPLES FOR FOLIATIONS

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1. Codimension > 1

Classical h-principles hold in codimension > 1.

- The open case was first discussed in Haefliger's *Feuilletages sur les varits ouvertes* (http://www.sciencedirect.com/science/article/pii/0040938370900406).
- The closed case was proven by Thurston: *The theory of foliations of codimension greater than one* (http://link.springer.com/article/10.1007%2FBF02566730).
- Yasha has a proof using wrinkling in *Wrinkling of smooth mappings III, Foliations of codimension greater than one* (http://www-users.mat.umk.pl/~tmna/files/v11n2-09.pdf).
- A detailed proof in the special case of codimension 2 foliations on a 4-manifold appears in Mitsumatsu and Vogt's *Thurstons h-principle for 2-dimensional Foliations of Codimension Greater than One* (http://arxiv.org/pdf/1509.06881v1.pdf).

2. Codimension 1

It also holds in codimension 1.

- Proved by Thurston in *Existence of Codimension-One Foliations* (https://www.jstor.org/stable/1971047). The special case of dimension 3 is done in Thurston's *A local construction of foliations for three-manifolds* and supposedly easier.

3. Haefliger's classifying space

Formal foliations turn out to be sections of a bundle with fiber $B\Gamma_q^r$, called Haefliger's classifying space.

- The original reference is Haefliger's *Homotopy and integrability* (http://link.springer.com/chapter/10.1007%2FBFb0068615).
- Segal gave a different model more geometric model in *Classifying spaces related to foliations* (http://www.sciencedirect.com/science/article/pii/0040938378900046).

4. Mather-Thurston theory

The application of h-principles for foliations to the study of diffeomorphisms (or homeomorphism) groups is called Mather-Thurston theory. There does not seem to be a complete exposition of it.

- The input is information about the homology of compactly-supported diffeomorphism groups of $\mathbb{R}^n$. These theorems were proven in a sequence of papers by Mather: *Integrability in codimension 1* for $\mathbb{R}^1$ (http://link.springer.com/article/10.1007%2FBF02566122), *Loops and foliations* for $\mathbb{R}^n$, *The vanishing of the homology of certain groups of homeomorphisms* for homeomorphisms (http://www.sciencedirect.com/science/article/pii/004093837190022X).
- The general manifold case is due to Thurston, *Foliations and groups of diffeomorphisms* (http://www.ams.org/journals/bull/1974-80-02/S0002-9904-1974-13475-0/home.html).