MATH 4520 · Classical geometry and modern applications
Fall 2021
Draft syllabus

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What is geometry? Geometry is the study of the structure and shape of spaces, both abstract and real-world. It provides much of the framework for modern physics and touches many areas of mathematics. Any mathematics that uses the language of shape and distances or angles (e.g. "orthogonal", "isometry") is based in geometry.

Course format. This class has a unique and unusual structure!
On a typical week, one day (usually Monday) will be a standard lecture, like most of your other math classes. Another day (usually Wednesday) will be devoted to time to work on homework problems in groups, other activities, or occasionally a catch-up period for more lecture or presentation. The third day (usually Friday) will be a full period of students at the board. That means you and your peers will do the talking! You will be able to request particular problems to present the day before (via a google form) and I will make sure that everyone gets a chance to speak during the semester.

Learning outcomes. Mathematically, our goal is to understand fundamental geometric structures from the big three classical geometries (Euclidean, hyperbolic, projective) and how they relate to other areas of mathematics (linear and abstract algebra, number theory, etc.) and applications beyond mathematics. Skills you will develop include:
- Independent and group problem-solving skills through homework sets,
- Proof writing, explaining your thinking, and expository mathematical writing,
- Verbal/blackboard presentation skills, through student presentations, and
- Ability to explore, research, and discover, through problems and independent projects.

Prerequisites. This is a 4000-level course. You should have experience writing proofs and with abstract reasoning. The formal prerequisites are calculus and linear algebra; however, I strongly recommend having taken at least one 3000 level course. I expect a diverse range of backgrounds and interests among students in the class (and that's great!) but if you have concerns or questions about whether the class is at the right level for you, talk to me right away.

No textbook. There is no textbook for this course. I will assign some required readings from various sources (textbooks and articles), all of which will be available on canvas.

Problem sets. Homework sets will be assigned weekly, posted on the course Canvas site.
Please write clearly. There are resources on the course Canvas site and a homework template you can use (optionally) if you wish to type your homework using LaTeX.

Projects. At the end of each unit, we will have a short project-format homework instead of a problem set. This will involve doing a more in-depth exploration of a focused topic, and allow you to explore connections between geometry and other areas. I will provide project outlines, but I am open to other ideas if there is something specific you would like to pursue – talk to me early!

There is no exam, but rather a final project to be completed at the end of the semester. The final weeks of class will be devoted to student presentations, and a written component to this project due during exam week. A rubric and more details will be provided during the term.

Do your own work. You are encouraged to work with your peers on problem sets and will have designated class time for this. However, you must write up your own solutions. Set aside time for yourself to write a good copy of your solutions alone.

You are expected to follow Cornell's code of academic conduct. As such, plagiarism, copying, taking answers or parts of answers from books, notes, the internet, other people, etc, etc. is not tolerated and will result in a score of 0 for all people involved and may lead to an academic integrity hearing and further consequences.

Assessment
- Class participation 15%
  - attend class
  - participate in group work and in-class homework time
  - present homework problem solutions on presentation days, and listen to others presentations
- In-class prelim: Date TBA 20%
- Homework sets 30%
  - submit one problem (graded for completion) on gradescope before group-work day
  - submit full problem set on time, in class on the due date
- Small projects (two) 15% total
- Final project + project presentation 20%

In the event of extended illness or absence during the semester, contact me as soon as possible so that we can work something out!

Accommodations. Please contact Cornell SDS if you need accommodations for class, assignments, or the prelim. They will send me a letter. Please also feel free to reach out to me by e-mail or set up an appointment to chat.

Topic overview.
1. Euclidean geometry.
Our goal is to understand the relationship between the kind of Euclidean geometry proofs you might have seen in high school and the advanced mathematics in abstract algebra, field theory, and arithmetic.

2. Hyperbolic geometry.
Hyperbolic geometry is important in mathematics (in topology and differential geometry) and in physics: hyperbolic space appears in the geometry of relativistic spacetime. We'll understand this space through both a *synthetic* (axiomatic/constructive) approach and an *analytic* (compute in coordinates) approach, and see the connections to some of the areas mentioned above.

3. Projective geometry.
This is the geometry of cameras and vision and computer vision. Since far away objects appear smaller (without actually being smaller) in projective geometry there is no definition of length or size or measurement of angle! Projective geometry also has a close relationship with algebra, and if you've read the (very fun) book *How not to be wrong* by J. Ellenberg, you'll know that you can also use projective geometry to win the lottery.

Schedule. See the separate handout for a draft of a day-by-day schedule.