ERRATUM FOR "FROBENIUS MODULES AND DE JONG'S THEOREM"

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Laurent Berger has pointed that out that [1, Lemma 5.6] is incorrect as stated, and is in fact contradicted by the first example at the end of [1, §5]: for $\sigma(t) = (1+t)^p - 1$, one may take $g = 1$, $x = 1$, $y = \log(1+t)$ to obtain an example of elements $x, y \in \mathcal{R}$ with $x^\sigma = gx$, $y^\sigma = gpx$. In the rest of this document, we explain the error in the proof of [1, Lemma 5.6], give a corrected statement and proof, and correct the proof of [1, Theorem 3.4]; the upshot is that all other results of [1] remain unaffected.

The error in the proof of [1, Lemma 5.6] is in the equation $a/c = hu(r/s)$ just above [1, (5.7)]. Here, a/c is a rational function of t times a unit of Ω, and the erroneous claim is that it can be factored as $hu(r/s)$ where $h \in W[1/p]$, u is a principal unit of Ω, and r, s are polynomials in t with constant coefficient. The correct formula should be $a/c = hu(r/s)t^e$ where h, u, r, s are as before and e is an integer; this changes [1, (5.7)] to

$$bs^\sigma r = p^\ell (dr^\sigma s) \frac{(hu)^\sigma}{hu} \left(\frac{t^\sigma}{t} \right)^e. \tag{1}$$

This equation no longer implies that $\ell = 0$; rather, we have $\ell = -ev$ where v is the p-adic valuation of the constant coefficient of t^σ/t. (In particular, if σ is a Frobenius lift for which $v = 0$, such as the standard lift, then the lemma remains correct as stated.)

We now give a corrected statement.

Lemma. *Choose $x, y \in \mathcal{R}$ nonzero such that $x^\sigma = gx$, $y^\sigma = gp^\ell y$ for some $g \in \Gamma_c[\frac{1}{p}]$ and some nonnegative integer ℓ. Then y is divisible by x in $\mathcal{R}$.*

Proof. If $\ell = 0$, then the proof of [1, Lemma 5.6] as written suffices to show that $y/x \in W[\frac{1}{p}]^*$. We thus assume that $\ell > 0$ hereafter; this forces $v > 0$.

Rewrite (1) as

$$\frac{d}{b} = \left(\frac{u}{u^\sigma} \right) \left(\frac{r/s}{(r/s)^\sigma} \right) w, \qquad w = p^{-\ell} \frac{h^\sigma}{h} \left(\frac{t^\sigma}{t} \right)^{\ell/v},$$

noting that $w \in \Omega[\frac{1}{p}]$ has constant coefficient 1. Then apply σ repeatedly to obtain, for each nonnegative integer n,

$$\prod_{i=0}^{n-1} \frac{d^{\sigma^i}}{b^{\sigma^i}} = \frac{u}{u^{\sigma^n}} \frac{r/s}{(r/s)^{\sigma^n}} \prod_{i=0}^{n-1} w^{\sigma^i}.$$

By Remark 4.5, we may take limits to obtain

$$\frac{y/c}{x/a} = u \frac{r}{s} z, \qquad z = \prod_{i=0}^{\infty} w^{\sigma^i} \in \mathcal{R};$$

this proves the claim. □

We now correct the proof of [1, Theorem 3.4], starting from the sentence "Assume without loss of generality that $h_1 \neq 0$." Here, we further assume that ℓ_1 is maximal among indices i for which $h_i \neq 0$. As in the original text, we have $F\mathbf{w} = g\mathbf{w}$ for some $g \in \Gamma_c$ and $h_i^{\sigma^m} = gp^{-\ell_i}h_i$ for each i for which $h_i \neq 0$; we also have that the h_i generate the unit ideal in $\mathcal{R}$. Comparing h_i with h_1 using the new lemma, we deduce that h_i is divisible by h_1 in $\mathcal{R}$; consequently, h_1 is a unit in $\mathcal{R}$, and the proof continues as in the original text.

References

[1] K.S. Kedlaya, Frobenius modules and de Jong's theorem, *Math. Res. Lett.* **12** (2005), 303–320.