

Quiz 9 - Calculus 1A
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Answer as many questions as you can in 25 minutes.

1. (8 points) Let $f(x) = (1+x)^{\frac{1}{x}}$. Compute $\lim_{x \rightarrow 0} f(x)$.

Hint: Take natural logarithm (base e) and then apply l'Hôpital's rule.

2. In this problem you will sketch the graph of the following function:

$$y = f(x) = \frac{2x^3 + x^2 + 1}{x^2 + 1}$$

- 2a. (1 point) What is the domain of $f(x)$?
 2b. (1 point) What is the y -intercept?
 2c. (1 point) Using long-division show that

$$f(x) = 2x + 1 + \frac{-2x}{x^2 + 1}$$

- 2d. (1 point) Describe all of the asymptotes (horizontal, vertical, slant), if any.

Hint: Determining asymptotes requires that you compute the behavior of $f(x)$ as $x \rightarrow \pm\infty$.

- 2e. (1 point) As $x \rightarrow \infty$ will $f(x)$ approach its asymptote from above or below (explain)?
 2f. (1 point) As $x \rightarrow -\infty$ how will $f(x)$ approach its asymptote (explain)?
 2g. (1 point) Is $f(x)$ an even function, an odd function, or neither?
 2h. (1 point) By noticing that

$$f(x) = \frac{2x^3}{x^2 + 1} + \frac{x^2 + 1}{x^2 + 1} = \frac{2x^3}{x^2 + 1} + 1$$

can you state anything about the symmetry of $f(x)$?

- 2i. (1 point) Compute the derivative $f'(x)$.

Suggestion: Use the simplified form from (2c), or (2h).

- 2j. (1 point) Where is $f'(x)$ positive?
 2k. (1 point) Where is $f'(x) = 0$, and is this a local max, local min, or neither?
 2l. (1 point) $f(x)$ has exactly 3 inflection points. Sketch the graph of $f(x)$.

Extra Credit. (1 point) In problem (2), without computing the second derivative $f''(x)$, can you explain why $f(x)$ must have at least 3 inflection points?

Extra Credit. (1 point) If one computes the limit in problem (1) by taking a logarithm base 2, how does the computation proceed?