Quiz 7 - Calculus 1A

October 18, 2004

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Answer as many questions as you can in 30 minutes.

In problems #1 and #2 concerning linear approximations, you should use the formula

$$f(a+\Delta x) \approx f(a) + \Delta x \cdot f'(a)$$

When working each problem you must clearly specify the following data:

$$\begin{aligned} f(x) &= \cdots \\ a &= \cdots \\ \Delta x &= \cdots \end{aligned}$$

1. (4 *points*) Estimate the value of $\sqrt{99.8}$

2a. (3 *points*) Estimate the value of $\ln(1.05)$
2b. (1 *point*) Justify this result by drawing the graph of $\ln x$ and its tangent line at $(1, 0)$.

In problems #3 and #4 identify the (x, y)-values of the points at which $y = f(x)$ has a local maximum, local minimum, absolute maximum, absolute minimum.

3. (4 *points*) $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the closed interval $[-2, 3]$

4. (4 *points*) $f(x) = \sin x + \cos x$ on the closed interval $[0, \frac{\pi}{3}]$

5. (4 *points*) Fill in the blanks in the following paragraph:

We say that a function $f(x)$ is *continuous* at $x = a$ if it has the property that

$$\text{small values of } \Delta x \Longrightarrow f(a + \boxed{\text{5a}}) \approx f(a)$$

In precise mathematical language this is expressed as:

$$\lim_{\Delta x \to 0} f(a+\Delta x) = f(\boxed{\text{5b}})$$

If, in addition, $f(x)$ has the property that it is *differentiable* at $x = a$, then we can say more; namely we can *quantify* how close $f(a + \Delta x)$ is to $f(a)$ for small values of Δx:

$$f(a + \boxed{\text{5c}}) \approx f(a) + \Delta x \cdot f'(\boxed{\text{5d}})$$

The basis for this approximate formula is that

$$\begin{aligned} f'(a) &= \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x} && \text{(by definition of derivative)} \\ &\approx \frac{f(a+\Delta x) - f(a)}{\Delta x} && \text{(for small values of } \Delta x) \end{aligned}$$

so after multiplying both sides by Δx we get:

$$\boxed{\text{5e}} \cdot f'(a) \approx f(a+\Delta x) - f(a) \qquad \text{(for small values of } \Delta x)$$

and so finally after adding $f(a)$ to both sides we get:

$$\boxed{\text{5f}} + (\Delta x) \cdot f'(a) \approx f(a+\Delta x) \qquad \text{(for small values of } \Delta x)$$