Quiz 6 Comments - Calculus 1A

October 11, 2004

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Following each solution is a list of common mistakes - those marked with an asterisk are considered substantive, others are more stylistic.

1. (4 *points*) Find the equation of the tangent line to the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at the point $P(-3\sqrt{3}, 1)$. Solution: $(y-1) = \frac{1}{\sqrt{3}}(x - 3\sqrt{3})$.

1. * losing a '-' sign when solving for y' explicitly
2. * losing a '-' sign when when taking cube-root of $(-3\sqrt{3})$
3. failing to notice $(3\sqrt{3}) = (\sqrt{3}\sqrt{3}\sqrt{3})$ so $\sqrt[3]{3\sqrt{3}} = \sqrt{3}$
4. (or if you prefer exponents) $(3\sqrt{3}) = (3^1 \cdot 3^{\frac{1}{2}}) = 3^{\frac{3}{2}}$ so $(3\sqrt{3})^{\frac{1}{3}} = 3^{\frac{1}{2}}$
5. unnecessarily solving for b to put equation in the form $y = mx + b$

2. (4 *points*) Evaluate the derivative of $y = \cot^{-1}(t) + \cot^{-1}(1/t)$. Solution: $y' = 0$.

1. * failing to use chain rule to introduce the factor $(\frac{1}{t})' = -\frac{1}{t^2}$
2. * failing to simplify the terms adding up to 0
3. easy solution is to notice $\cot^{-1}(t) + \cot^{-1}(1/t) = \cot^{-1}(t) + \tan^{-1}(t) = \frac{\pi}{2}$ =constant.

3. Assume the motion of an object is described by the equation $s(t) = 2t^3 - 9t^2$ where s is measured in meters, and t is measured in seconds.
(a) (2 *points*) Find the time at which the acceleration is 0. Solution: $t = \frac{3}{2}$ seconds
(b) (2 *points*) Find the displacement and velocity at the these times.
Solution: $s(t = \frac{3}{2}) = -\frac{27}{2}$ meters; $v(t = \frac{3}{2}) = -\frac{27}{2}$ meters/second

1. arithmetic errors
2. failure to indicate units such as meters/second

4. (4 *points*) Using logarithmic differentiation evaluate the derivative of $y = x^{\frac{1}{x}}$
Solution: $y' = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$

1. * failing to use chain rule to introduce the factor $(\frac{1}{x})' = -\frac{1}{x^2}$
2. * losing a '-' sign when introducing $(\frac{1}{x})' = -\frac{1}{x^2}$

5. (4 *points*) Evaluate the derivative of $y = \ln\big(\sinh(t)\big)$ Solution: $y' = \coth(t)$

1. failing to simplify $\frac{\cosh t}{\sinh t} = \coth(t)$