Quiz 4 - Calculus 1A
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Solutions

1. (3 *points*) In the following table, match each description of the 3 ways in which a function can fail to be differentiable at a point with the corresponding definition in the middle column and also with the corresponding example in the right column.

$f(x)$ has a corner at 0	$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$	$f(x) = \sqrt[3]{x}$
$f(x)$ has a vertical tangent at 0	$\lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x)$	$f(x) = \lvert x\rvert$
$f(x)$ fails to be continuous at 0	$f'(0) = \pm\infty$	$f(x) = \frac{\lvert x\rvert}{x}$

Solution:

$$\begin{array}{rcccl} f(x) \text{ has a corner at } 0 & \longleftrightarrow & \lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x) & \longleftrightarrow & f(x) = |x| \\ f(x) \text{ has a vertical tangent at } 0 & \longleftrightarrow & f'(0) = \pm\infty & \longleftrightarrow & f(x) = \sqrt[3]{x} \\ f(x) \text{ fails to be continuous at } 0 & \longleftrightarrow & \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x) & \longleftrightarrow & f(x) = \frac{|x|}{x} \end{array}$$

2a. (1 *point*) Given two differentiable functions $f(x)$ and $g(x)$, give the formula for the derivative of $\frac{f(x)}{g(x)}$ (valid for those x at which $g(x) \neq 0$).
Solution:

$$\left(\frac{f}{g}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

2b. (1 *point*) Apply the above formula to the special case in which $g(x) = 1$.
Solution: Since $g(x) = 1$, also $g'(x) = 0$, so:

$$\left(\frac{f}{1}\right)' = \frac{(1)\cdot f'(x) - f(x)\cdot(0)}{(1)^2} = f'(x)$$

2c. (1 *point*) Apply the above formula to the special case in which $f(x) = 1$.
Solution: Since $f(x) = 1$, also $f'(x) = 0$, so:

$$\left(\frac{1}{g}\right)' = \frac{g(x)\cdot(0) - (1)\cdot g'(x)}{g^2(x)} = \frac{-g'(x)}{g^2(x)}$$

2d. (1 *point*) Specialize to the case in which $f(x) = 1$ and $g(x) = x + 4$.
Solution: Using (2c):

$$\left(\frac{1}{(x+4)}\right)' = \frac{-(x+4)'}{(x+4)^2)} = \frac{-1}{(x+4)^2}$$

2e. (1 *point*) Specialize to the case in which $f(x) = x$ and $g(x) = x + 4$.
Solution: Using (2a):

$$\left(\frac{x}{x+4}\right)' = \frac{(x+4)(x)' - (x)(x+4)'}{(x+4)^2} = \frac{(x+4)(1) - (x)(1)}{(x+4)^2} = \frac{4}{(x+4)^2}$$

3. (5 *points*) Compute the derivative of

$$f(x) = \frac{x^2 + 4xe^x}{\sqrt{x}}$$

Solution: Absorb the denominator into the numerator:

$$f(x) = x^{-\frac{1}{2}}(x^2 + 4xe^x) = x^{\frac{3}{2}} + 4x^{\frac{1}{2}}e^x$$

Now by power rule and product rule:

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 4\left[(x^{\frac{1}{2}})'e^x + (x^{\frac{1}{2}})(e^x)'\right]$$

or:

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \left[2x^{-\frac{1}{2}}e^x + 4x^{\frac{1}{2}}e^x\right]$$

4. (5 *points*) Find the equation of the tangent line to the graph of the following function at the point $(0, 2)$:

$$f(x) = \frac{2e^x}{1+x}$$

Solution: First compute the derivative using quotient rule:

$$\begin{aligned} f'(x) &= 2\frac{(1+x)(e^x)' - (e^x)(1+x)'}{(1+x)^2} \\ &= 2\frac{(1+x)(e^x) - (e^x)(1)}{(1+x)^2} \\ &= 2\frac{xe^x}{(1+x)^2} \end{aligned}$$

Now evaluating the derivative at the point in question, namely $(x_0 = 0, y_0 = 2)$, we get

$$f'(0) = 2\frac{(0)e^0}{(1+(0))^2} = 0$$

Finally, using point-slope formula $(Y - y_0) = m(X - x_0)$ with $m = f'(x_0) = f'(0)$:

$$(Y - 2) = (0)(X - 0) \text{ or } Y = 2$$

5. (2 *points*) Determine the vertical and horizontal asymptotes of the graph of the following function:

$$f(x) = \frac{x}{x+4}$$

Solutions: See "Asymptote Problem" on Web-page. Note that this problem does **not** involve computing the derivative $f'(x)$.