

## Quiz 15 - Calculus 1A

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Answer as many questions as you can (20 + 5 points total) in 30 minutes.

1. (10 *points*) Find the area of the region bounded above by  $y = e^x$ , bounded below by  $y = x$ , and bounded on the sides by  $x = 0$  and  $x = 1$ .
2. (5 *points*) Show that the volume of a right circular cone with height  $h$  and radius  $r$  is given by  $\frac{1}{3}\pi r^2 h$ .
3. (5 *points*) Find the average value of the function  $f(x) = xe^{-x^2}$  on the interval  $[0, 5]$ .

Extra Credit. (1 *point*) Show that the volume of a sphere of radius  $r$  is given by  $\frac{4}{3}\pi r^3$ .

Extra Credit. (1 *point*) Find the average value of the function  $(x - 3)^2$  on the interval  $[2, 5]$ .

Extra Credit. (1 *point*) Find the volume of the solid obtained by rotating about the  $x$ -axis the region bounded by the following curves:

$$y = \sqrt{x}, \quad y = 0, \quad x = 9$$

*Hint: Use integration of  $A(x) = \pi y(x)^2$  with respect to  $x$  from  $x = 0$  to  $x = 9$ .*

Extra Credit. (1 *point*) Find the volume of the solid obtained by rotating about the  $y$ -axis the region bounded by the following curves:

$$y = x^2, \quad y = 0, \quad x = 3$$

*Hint: Use integration of  $2\pi x \cdot y(x)$  with respect to  $x$  from  $x = 0$  to  $x = 3$ .  
Alternatively, you may leverage your answer to previous problem,*

Extra Credit. (1 *point*) Find the area of the region enclosed between the curves  $y = \frac{1}{x}$ ,  $y = \frac{1}{x^2}$ ,  $x = 0.5$ , and  $x = 2$ .

*Hint: Determine points of intersection and break problem up into two integrations with respect to  $x$ .*