Calculus 1A

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Some Solutions to Homework 6

(§3.6, #25) Find the equation of the tangent line to the ellipse $x^2 + xy + y^2 = 3$ at the point P(1, 1).
Solution: Differentiate implicitly to get (implicit) equation for y':

$$\begin{aligned} x^2 + xy + y^2 = 3 &\implies (x^2)' + (xy)' + (y^2)' = (1)' \\ &\implies (2x) + (xy' + x'y) + (2yy') = 0 \\ &\implies (2x) + (xy' + y) + (2yy') = 0 \end{aligned}$$

Now specialize to $x = 1$ and $y = 1$ to get (implicit) equation for y' at P.

$$\begin{aligned} \text{(evaluate above at P}(x = 1, y = 1)) &\implies (2) + (y'_P + 1) + (2y'_P) = 0 \\ &\implies 3 + 3y'_P = 0 \\ \text{(solve to get explicit value of } y' \text{ at P)} &\implies y'_P = -1 \end{aligned}$$

Finally, using equation $(y - y_0) = y'_P(x - x_0)$ gives:

$$(y - 1) = (-1)(x - 1)$$

(§3.6, #49) Evaluate the derivative of $y = \cos^{-1}(e^{2x})$.
Solution:

$$\begin{aligned} \text{since } (\cos^{-1} z)' = (\tfrac{-1}{\sqrt{1-z^2}})(z)' &\implies y' = (\frac{-1}{\sqrt{1 - (e^{2x})^2}})(e^{2x})' \\ \text{since } (e^z)' = (e^z)(z)' &\implies y' = (\frac{-1}{\sqrt{1 - e^{4x}}})(e^{2x})(2x)' \\ &\implies y' = \frac{2e^{2x}}{\sqrt{1 - e^{4x}}} \end{aligned}$$

(§3.7, #47) Assume the motion of an object is described by the equation $s(t) = t^4 - 4t^3 + 2$ where s is measured in meters, and t is measured in seconds.
(a) Find the time at which the acceleration is 0.
Solution:

$$\begin{aligned} s(t) = t^4 - 4t^3 + 2 &\implies v(t) = s'(t) = 4t^3 - 12t^2 \\ &\implies a(t) = v'(t) = s''(t) = 12t^2 - 24t \end{aligned}$$

so

$$\begin{aligned} a(t)=0 &\implies 12t^2-24t=0 \\ &\implies 12t(t-2)=0 \\ &\implies t=0 \text{ or } t=2 \text{ seconds} \end{aligned}$$

(b) Find the displacement and velocity at the these times.
Solution:

$$\begin{aligned} s(t=0) = 0^4 - 4\cdot 0^3 + 2 = 2 \text{ meters} \\ v(t=0) = 4\cdot 0^3 - 12\cdot 0^2 = 0 \text{ meters/second} \\ s(t=2) = 2^4 - 4\cdot 2^3 + 2 = 16 - 32 + 2 = -14 \text{ meters} \\ v(t=2) = 4\cdot 2^3 - 12\cdot 2^2 = 32 - 48 = -16 \text{ meters/second} \end{aligned}$$

(§3.8, #43) Using logarithmic differentiation evaluate the derivative of $y = \big(\ln x\big)^x$
Solution:

$$\begin{aligned} y = \big(\ln x\big)^x &\implies \ln(y) = x\big(\ln(\ln x)\big) \\ \text{differentiating and using } (\ln x)' = \tfrac{1}{x} &\implies \big(\frac{1}{y}\big)y' = x\big(\ln(\ln x)\big)' + (x)'\big(\ln(\ln x)\big) \\ &\implies \big(\frac{1}{y}\big)y' = x\big(\frac{1}{\ln x}\big)(\ln x)' + \big(\ln(\ln x)\big) \\ &\implies \big(\frac{1}{y}\big)y' = x\big(\frac{1}{\ln x}\big)\frac{1}{x} + \big(\ln(\ln x)\big) \\ &\implies \big(\frac{1}{y}\big)y' = \frac{1}{\ln x} + \ln(\ln x) \\ &\implies y' = \big(y\big)\big[\frac{1}{\ln x} + \ln(\ln x)\big] \\ \text{substituting original expression for } y(x) &\implies y' = \big(\ln x\big)^x\big[\frac{1}{\ln x} + \ln(\ln x)\big] \end{aligned}$$

(§3.9, #41) Evaluate the derivative of $y = e^{\cosh 3x}$
Solution:

$$\begin{aligned} y' &= \big(e^{\cosh 3x}\big)\big(\cosh 3x\big)' \\ (\text{since } (\cosh z)' = (\sinh z)z') &= \big(e^{\cosh 3x}\big)\big(\sinh 3x\big)(3x)' \\ &= \big(e^{\cosh 3x}\big)\big(\sinh 3x\big)(3) \\ &= 3(\sinh 3x)(e^{\cosh 3x}) \end{aligned}$$

Alternate Solution: Use logarithmic differentiation.