

Common Errors on Week 2 Homework

1. (§2.4, #3) To make $|\frac{1}{x} - .5| < .2$ we argue as follows:

$$\begin{aligned}
 |\frac{1}{x} - .5| < .2 &\iff - .2 < \frac{1}{x} - .5 < +.2 \\
 &\iff - .2 + .5 < \frac{1}{x} < +.2 + .5 \\
 &\iff (.3) < \frac{1}{x} < (.7) \\
 (z \rightarrow z^{-1} \text{ is decreasing function on } \mathbf{R}^+) &\iff (.3)^{-1} > x > (.7)^{-1} \\
 &\iff \frac{10}{3} > x > \frac{10}{7} \\
 &\iff \frac{10}{3} - 2 > x - 2 > \frac{10}{7} - 2 \\
 &\iff \frac{4}{3} > x - 2 > -\frac{4}{7} \\
 (\text{note one-way implication since } \frac{4}{7} < \frac{4}{3}) &\iff \frac{4}{7} > x - 2 > -\frac{4}{7} \\
 &\iff |x - 2| < \frac{4}{7}
 \end{aligned}$$

In other words, $|x - 2| < \frac{4}{7} \implies |\frac{1}{x} - .5| < .2$

2. (§2.4, #5) To make $|\sqrt{x} - 2| < .4$ we argue as follows:

$$\begin{aligned}
 |\sqrt{x} - 2| < .4 &\iff - .4 < \sqrt{x} - 2 < +.4 \\
 &\iff - .4 + 2 < \sqrt{x} < +.4 + 2 \\
 &\iff 1.6 < \sqrt{x} < 2.4 \\
 (z \rightarrow z^2 \text{ is increasing function on } \mathbf{R}^+) &\iff (1.6)^2 < x < (2.4)^2 \\
 &\iff 2.56 < x < 5.76 \\
 &\iff 2.56 - 4 < x - 4 < 5.76 - 4 \\
 &\iff -1.44 < x - 4 < 1.76 \\
 (\text{note one-way implication since } 1.44 < 1.76) &\iff -1.44 < x - 4 < 1.44 \\
 &\iff |x - 4| < 1.44
 \end{aligned}$$

In other words, $|x - 4| < 1.44 \implies |\sqrt{x} - 2| < .4$