UCB Math 110, Spring 2011: Homework 7
Solutions to Graded Problems

6.1.1(g) 2 points. False. Consider $\mathbb{R}^2$ with the standard inner product. We see

$$\left\langle \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\rangle,$$

a counter example.

6.1.16(b) 4 points. No this is not an inner product. Consider

$$f(t) = \begin{cases} 0 & 0 \leq t \leq \frac{1}{2} \\ t - \frac{1}{2} & \frac{1}{2} < t \leq 1 \end{cases}.$$

It is easy to see that $\langle f(t), f(t) \rangle = 0$ even though $f \neq 0$, violating the definition of inner product.

6.2.15(a) 4 points. Since $\{v_1, \ldots, v_n\}$ is an orthonormal basis we see that

$$x = \langle x, v_1 \rangle v_1 + \langle x, v_2 \rangle v_2 + \ldots + \langle x, v_n \rangle v_n = \sum_{i=1}^{n} \langle x, v_i \rangle v_i. \tag{1}$$

Thus

$$\begin{aligned} \langle x, y \rangle &= \left\langle \sum_{i=1}^{n} \langle x, v_i \rangle v_i, y \right\rangle \\ &= \sum_{i=1}^{n} \langle x, v_i \rangle \langle v_i, y \rangle \\ &= \sum_{i=1}^{n} \langle x, v_i \rangle \overline{\langle y, v_i \rangle} \end{aligned}$$

6.2.15(b) 2 points. Let $\beta = \{v_1, \ldots, v_n\}$. By equation (1) we see that

$$[x]_\beta = \begin{bmatrix} \langle x, v_1 \rangle \\ \vdots \\ \langle x, v_n \rangle \end{bmatrix}, \quad [y]_\beta = \begin{bmatrix} \langle y, v_1 \rangle \\ \vdots \\ \langle y, v_n \rangle \end{bmatrix}.$$

Thus we get

$$\begin{aligned} \langle [x]_\beta, [y]_\beta \rangle &= \sum_{i=1}^{n} \langle x, v_i \rangle \overline{\langle y, v_i \rangle} \\ &= \langle x, y \rangle. \end{aligned}$$