Note Title 3/6/2011

5.1.1.c (T) View $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in M_2(\mathbb{R})$

then $P_A(\lambda) = \lambda^2 + 1$ and thus A

has no e-values ✓

5.1.4.i Using the standard basis for $M_2(\mathbb{R})$

we see that $[T] = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

It follows that

$$P_T(\lambda) = (\lambda+1)^2(\lambda-1)^2$$

Since $[T] - I = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$ basis for null space $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

and $[T] + I = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ basis for null space $\left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

We see that $\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \right\}$

is a basis of e-vectors for T ✓

5.2.1.i (F)

Let $V = \mathbb{R}^2$, $W_1 = \{(x,y) \mid y = x\}$, $W_2 = \{(x,y) \mid y = -x\}$

$W_3 = \{(y,x) \mid x = 0\}$

Then $W_1 + W_2 + W_3 = \mathbb{R}^2$ and $W_i \cap W_j = \{0\}$

but since $W_i + W_j = \mathbb{R}^2$

$W_1 + W_2 + W_3$ is not a direct sum. ✓

5.2.18a let β be a basis $\ni$ $[T]_\beta$ and $[U]_\beta$ are diagonal

Then $[UT]_\beta = [U]_\beta [T]_\beta = [T]_\beta [U]_\beta = [TU]_\beta$

So $UT = TU$ since $[\ ]_\beta$ is 1-1 ✓