UCB Math 110, Spring 2011: Homework 1
Solutions to Graded Problems

1.2.1(b) False. Let $0, 0' \in V$ be two zero vectors. Then $0 = 0 + 0' = 0'$.

1.3.23(a) First we show that $W_1 + W_2$ is a subspace. As both W_i is a subspace of V for all i, we have $0 \in W_i$. Thus $0 + 0 = 0 \in W_1 + W_2$. Let $x, y \in W_1 + W_2$ and $c \in F$. Then $x = x_1 + x_2$ and $y = y_1 + y_2$ for some $x_i, y_i \in W_i$ for all i. A simple calculation gives that

$$x + cy = (x_1 + cy_1) + (x_2 + cy_2).$$

Once again W_i is a subspace of V for all i giving that $x_i + cy_i \in W_i$. Thus $x + cy \in W_1 + W_2$, implying that $W_1 + W_2$ is a subspace of V. Let $x \in W_1$. As $0 \in W_2$, we get that $x + 0 = x \in W_1 + W_2$. Hence $W_1 \subset W_1 + W_2$. Similarly $W_2 \subset W_1 + W_2$.

1.5.1(d) False. Counter Example: Consider the set $\{0\}$ which is linearly dependant. $\emptyset \subset \{0\}$ is linearly independant.

1.6.22 The condition is that $W_1 \subset W_2$, thus we must prove that "$\dim(W_1 \cap W_2) = \dim(W_1)$ if and only if $W_1 \subset W_2$."
Let $\dim(W_1 \cap W_2) = \dim(W_1)$. As $W_1 \cap W_2 \subset W_1$, we see that our assumption gives that $W_1 \cap W_2 = W_1$. Thus $W_1 = W_1 \cap W_2 \subset W_2$.
Conversely let $W_1 \subset W_2$. Thus $W_1 \cap W_2 = W_1$ giving that $\dim(W_1 \cap W_2) = \dim(W_1)$.