

Midterm #2 110 Spring 11

Solutions

1. Charac poly is $p(z) = (1-z)^2(3-z)$

thus the eigenvalues are in $\{1, 3\}$

The eigenvectors with eigenvalue 1 satisfy

$$\begin{pmatrix} x_2 \\ 2x_2 \\ 2x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \text{ There is a one}$$

dimensional space of eigenvectors, spanned

by $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ for an eigenvalue of multiplicity 2

Answers is no!

$$2. Ax = 0 \Leftrightarrow A^*Ax = 0$$

if $Ax = 0$ then trivially $A^*Ax = 0$

if $A^*Ax = 0$ then $\langle A^*Ax, x \rangle = \langle Ax, Ax \rangle = \|Ax\|^2 = 0$
and then $Ax = 0$

Thus nullity of A & A^*A agree and by
the dimension theorem $\text{rank } A = \text{rank } A^*A$

3. If T^* is not 1-1, there is $y \neq 0$: $T^*y = 0$

$$\langle Tx, y \rangle = \langle x, T^*y \rangle = 0 \text{ for all } x$$

this implies that $y \perp \text{range of } T$ and T is
not onto

4. T, F, F, T