Solutions

6.5 1b) $\boxed{F}$ Consider $T: \mathbb{R}^2 \longmapsto \mathbb{R}^2$ T 90° rotation.

Then in the standard basis $[T] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ So cols are orthonormal

$\Rightarrow$ T is orthogonal, however T is not diagonalizable since it has no eigenvalues in $\mathbb{R}$

6.5 1h) $\boxed{F}$ Consider $T: \mathbb{C}^2 \longrightarrow \mathbb{C}^2$

$[T] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ 1 is the only eigenvalue of T, but the cols of $[T]$ are not orthonormal so T is not unitary or orthogonal.

6.6. 5a) Suppose $T: V \longmapsto V$ is an orthogonal Projection, where V is a finite dimensional inner product space

Then $V = R(T) \oplus N(T)$ & $(N(T))^{\perp} = R(T)$

So if $x \in V$ $x = r + n$, $n \in N(T)$, $r \in R(T)$, $T(x) = r$

& $\|x\|^2 = \langle r+n, r+n \rangle = \|r\|^2 + \langle r, n \rangle + \langle n, r \rangle + \|n\|^2$

$= \|r\|^2 + \|n\|^2 \geq \|r\|^2 = \langle T(x), T(x) \rangle = \|T(x)\|^2$

$\Rightarrow \|T(x)\| \leq \|x\| \quad \forall x \in V.$

If $\|T(x)\| = \|x\|$ $\forall x \in V$ then T is unitary so invertible

Thus $Id = T^{-1}T = T^{-1}T^2 = T$

Consider $T: \mathbb{R}^2 \longmapsto \mathbb{R}^2$ $[T] = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$

T is a projection since $[T]^2 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$

but $\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = T\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

so $\left\| T\begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\| = 2 > \sqrt{2} = \left\| \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\| \square$