HW 6 SOLUTIONS

5.3 1(g) $\boxed{T}$ $(1,1,\dots,1)$ is an eigenvector with eigenvalue 1 of the matrix A^T since the rows of A^T sum to 1.
$\det(A-\lambda Id) = \det(A^T - \lambda Id)$ so A also has 1 as an eigenvalue.

5.3.2(i) $P_A(\lambda) = (\frac{1}{2}-\lambda)(1+\lambda^2) \Rightarrow \lambda = \frac{1}{2}, i, -i$ the limit does not exist
Since $|i| = 1$ but $i \neq 1$
(by Thm 5.13)

5.4(c) F. Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the identity.
$e_1 \neq 2e_2$ but the T-cyclic subspaces they generate both equal the x-axis $= \{(x,0) : x \in \mathbb{R}\}$

5.4.17. claim: $\{Id, \dots, A^{n-1}\}$ spans $W := \mathrm{Span}\{Id, A, A^2, A^3, \dots\}$
Thus $\dim W \leq n$.

pf: Induction. We will show $A^k \in \mathrm{span}\{Id, \dots, A^{n-1}\}$ $k \geq n$.

Base case: By Cayley Hamilton, $P_A(A) = (-1)^n A^n + a_{n-1}A^{n-1} + \dots + a_1 A + a_0 Id = 0$

$\Rightarrow A^n = (-1)^{n+1} a_{n-1} A^{n-1} + \dots (-1)^{n+1} A + (-1)^{n+1} a_0 Id$

so $A^n \in \mathrm{Span}\{Id, \dots, A^{n-1}\}$

Inductive Step: Assume $A^k \in \mathrm{span}\{Id, \dots, A^{n-1}\}$ Then for some b_i

$A^{k+1} = A(A^k) = A(b_n A^{n-1} + \dots b_1 A + b_0)$

$= b_n A^n + \dots + b_1 A^2 + b_0 A = b_n((-1)^{n+1} a_{n-1} A^{n-1} + \dots + (-1)^{n+1} a_0 Id)$
$+ \dots + b_1 A^2 + b_0 A$ $\square$