

Let X^\dagger be the dagger nbhd of a smooth one dim. affinoid X over K . Let $P \in X^\dagger$ be a base point.. We want a notion $I(w)$ of the "integral of w from P ." and two towers of $A(X^\dagger)$ -modules F_n of locally analytic functions and D_n of locally analytic differentials such that

$$\begin{aligned} F_1 &= A(X^\dagger), D_1 = \Omega_K^1(X^\dagger) \\ F_{n+1} &= K + I(D_n)F_1 \\ D_{n+1} &= F_n D_1 \end{aligned}$$

Then $dF_n \subset D_n$. Let $C^n = D_n/dF_n$. For $w_1, \dots, w_n \in D_1$, define $w_1 \dots w_n \in D_{n-1}$ to be $I(w_1, \dots, w_{n-1})w_n$. Then, of course, $I(w_1 \dots w_n)$ will be an n -th iterated integral.

What we have to do is define I . On D_1 we can do what Kiran said. Then,

$$C^2 \cong C^1 \otimes C^1. \tag{1}$$

If $w_1 \dots w_n$ are representatives for a basis of C^1 ,. then $v_{i,j} := I(w_i)w_j$ are representatives for a basis of C^2 . if we have a lifting of Frobenius ϕ fixing P , then have an action on C^1 and because $\phi^*df = d\phi^*f$ and $\phi^*I(w) = I(\phi^*w)$ an action on C_2 compatible with (1). Let V be the vector (matrix) $(v_{i,j})$. Suppose

$$\phi^*V = MV + dW$$

where W is a vector with entries in F_2 , M is a matrix of constants and $W(P) = 0$. Then we want, $dI(V) = V$, $I(V)(P) = 0$ and

$$\phi^*I(V) = MI(V) + W.$$

Since the eigenvalues of M are products of pairs of egenvalues of ϕ on C^1 , this completely determines I on D_2 .

It is clear how to continue.

In fact, $F_\infty := \bigcup_n F_n$ is an $A(W)$ -algebra. To see this we need another way of characterizing I . Let $Q_n(T)$ be the characteristic polynomial of ϕ on C_n . Then if $v \in D_n$,

$$Q_n(\phi^*)v = dw,$$

for a unique $w \in F_n$, $w(P) = 0$. Then, of course,

$$Q_n(\phi^*)I(v) = w.$$

and this yields another characterization of I . It follows that

$$I(v_1)I(v_2) = I(v_1I(v_2) + I(v_1)v_2).$$

because

$$Q_{n+m}(T) = \prod_i [\alpha_i^{d_n} Q_n(T/\alpha_i);$$

where $Q_m(t) = \prod_{i=1}^{d_m} (T - \alpha_i)$.

Suppose $Q(T) = \sum_{i=1}^d a_i T^i$. Then

$$\beta^d Q(\phi^*/\beta)uv = \beta^d (Q(\phi^*)u)v + \sum_{i=0}^d a_i \sum_{j=0}^{i-1} \beta^{d-j-1} (\phi^*)^i u \cdot (\phi^* - \beta)\phi^j v$$

so

$$Q_{n+m}(\phi^*)(uv) \in Q(\phi^*)M \cdot N + M \cdot Q_n(\phi^*)N$$

where M is the span in F_m of $\{(\phi^*)^i u\}$ and N is the span in F_n of $\{(\phi^*)^i v\}$.