Name:

Midterm 2
Math 53, Summer 2009
July 31, 2009
Brown

Problem	Score
1	/10
2	/10
3	/10
4	/10
5	/5
6	/5
Total	/50

1. (10 pt) Let $\mathbf{F}(x, y, z) = y^2 z\mathbf{i} + z^2 x\mathbf{j} + z\mathbf{k}$
a)(3 pt)Compute $\nabla \times \mathbf{F}$

(b)(3 pt)Compute $\nabla \cdot \mathbf{F}$

c)(4 pt) Let $\mathcal{S}$ be the hemisphere $z \geq 0$, $x^2 + y^2 + z^2 = 1$. Compute $\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n}\, dS$.

2. (10 pt) Consider the function $f(x, y) = x^2 - 2x + 1 - y^2 + 2y - 1 + xy - y - x + 1$. Find the absolute maximum and minimum of f on the rectangle with vertices $(0, 0), (0, 2), (2, 0), (2, 2)$ as well as at which points the maximum and minimum are attained.

3. (10 pt) Let R be the rectangle in the plane with vertices $(1,0),(3,2),(2,3),(0,1)$ Compute $\iint_R (x-y)^4\,dA$.

4. (10 pt) Let $\mathcal{C}$ be the triangle $(2,0,0),(0,1,0),(0,0,1)$ oriented so that the points are traversed in the order they are given above. Let $\mathbf{F} = \langle e^{x^2}, x + y^2 + z, y + \sin z \rangle$. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

5. (5 pt) Mike and Ivan are going to a bar after a long week. They both plan to arrive at 5, but Ivan has a habit of being late. Mike arrives sometime between 5 and 5:30, with constant probability density $f(x) = 1/30$. Ivan arrives sometime after 5, with probability density $f(y) = (1/60)e^{-y/60}$. What is the probability that Ivan arrives before Mike?

6. (5 pt) Let $\mathbf{F} = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$. Let $\mathcal{C}$ be the curve parametrized by $\mathbf{r}(t) = \langle 10\cos 2t + \sin 5t, 10\sin 2t + \cos t \rangle, 0 \leq t \leq 2\pi$. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.