Mathematics 160 Final

December 19, 2008. Professor H. Wu

Your Name: _____

You have TWO pages to write your answer to each problem. Write legibly and clearly. You may use a calculator.

1. (20%) Give a brief history of the emergence of algebraic symbols. In your answer, try to supply approximate *dates*, and be sure to touch on the role played by each of the following: (a) the Greek mathematicians in the B.C. era, (b) Diophantus, (c) the Islam mathematicians from 800 A.D. to 1100 A.D., approximately, (d) the Italian mathematicians of the 1500's, (e) Viète, and (f) Descartes.

(Cont. from # 1.)

2. (20%) If someone tells you "Newton and Leibniz invented calculus", what would you say to set him straight? Make your case by citing as many facts (with dates) as you can manage, and by explaining what "calculus" is about.

(Cont. from # 2.)

3. (20%) Explain why each of the following mathematicians is important in the evolution of mathematics: (a) Euler. (b) Emmy Noether. (c) Fermat. (d) Abel. (e) Hilbert.

(Cont. from # 3.)

4. (20%) (a) State Cavalieri's Principle. When did he live? (b) Explain briefly why the principle is correct. (c) Give the names of some mathematicians who anticipated Cavalieri. (d) Compute the volume of the sphere using this principle.

(Cont. from # 4.)

5. (20%) The number 4 is clearly a solution of the cubic equation $x^3 - 15x = 4$. However, the Cardano formulas give the following as the three roots of this cubic:

$$\left(-\frac{5}{b}\right)-b, \qquad \omega^2\left(-\frac{5}{b}\right)-\omega b, \qquad \omega\left(-\frac{5}{b}\right)-\omega^2 b,$$

where b satisfies $b^3 = -2 + \sqrt{-121}$, and $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. How do you reconcile these two facts?

(Cont. from # 5.)