

Your name: _____

MATHEMATICS 160 MIDTERM
APRIL 4, 2002, PROF. WU

I. (30%) Write T (true) or F (false) in the blank space in front of each of the following assertions. You get +3 for each correct answer, 0 for no answer, and -3 for each incorrect answer.

___ (1) Chinese mathematicians around 200 A.D. were already solving systems of linear equations by the method of Gaussian elimination.

___ (2) Ptolemy made effective use of Apollonius' results on conics in the former's great treatise *Almagest* on astronomy.

___ (3) After the fall of Rome in 476 A.D., mathematics was dormant in Europe for almost a thousand years.

___ (4) The logarithm function was discovered as a byproduct of the search for ways to simplify numerical computations.

___ (5) Euclid's parallel postulate (Fifth Postulate) became controversial because people believed that given a point P not on a line ℓ , it should be possible to prove rather than assume the existence of a line parallel to ℓ and passing through P .

___ (6) With the help of Eudoxus' theory of proportions, Euclid succeeded in the last book of his *Elements* to compute the volume of a ball (interior of a sphere) in terms of the radius.

___ (7) The Babylonians around 1700 B.C. already knew how to prove the existence of an infinite number of Pythagorean triples.

___ (8) The Hindus and the Chinese before 1300 A.D. were handicapped in their study of polynomial equations because they did not know the Pascal triangle and the binomial theorem.

___ (9) Although the Babylonians essentially knew how to derive the quadratic formula, it was not until the 9th century A.D. that Islam mathematicians could write out the general quadratic formula for $ax^2 + bx + c = 0$, where a , b , and c are arbitrary real numbers.

2

____ (10) Diophantus inherited from Euclid the Greek tradition of proofs, and his work on "Diophantine equations" was the first to prove theorems in number theory.

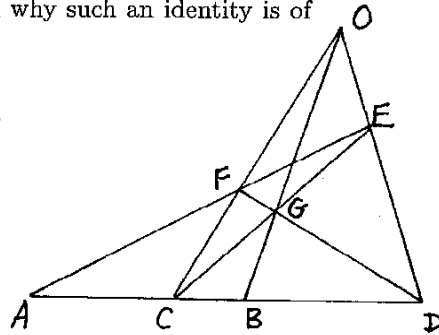
II. (10%) (1) Write a dozen lines or so on some of the significant names, dates, and events that are associated with *Alexandria*. (2) Give the approximate dates of Zhu Shijie (Chu Shi-Chieh), and in about five lines say what you know about his achievements.

III. (20%) (1) Prove the following identity of al-Haytham:

$$(n+1) \sum_{i=1}^n i^k = \sum_{i=1}^n i^{k+1} + \sum_{\nu=1}^n \left(\sum_{i=1}^{\nu} i^k \right),$$

where k, n are integers ≥ 1 . (2) Briefly explain why such an identity is of interest.

IV. (15%) Recall that four collinear points A, B, C , and D are said to be a *harmonic set* if $\frac{|AC|}{|BC|} \frac{|BD|}{|AD|} = 1$. Show that if C, D, E, F are four points, no three of which are collinear, and lines are drawn through them as shown, so that OG meets AD at B , then A, B, C , and D are a harmonic set. (You may make use of homework exercises for your proof.)



V. (15%) Given cubic equation $x^3 + ax + b = 0$ with a, b being complex numbers. (1) Show that if p, q are complex numbers which satisfy:

$$(*) \quad \begin{cases} 3pq & = a \\ p^3 - q^3 & = -b, \end{cases}$$

then $x = p - q$ would be a solution of $x^3 + ax + b = 0$. (2) Solve (*) to get one solution $x = p - q$ of $x^3 + ax + b = 0$ in terms of a and b .

VI. (10%) Show that if $\{a, b, c\}$ is a primitive Pythagorean triple with c as the hypotenuse, then c cannot be divisible by any prime $\equiv 3 \pmod{4}$.