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Stephen Smale: The Mathematician Who Broke the Dimension Barrier

Reviewed by Rob Kirby

Stephen Smale: The Mathematician Who Broke the Dimension Barrier
Steve Batterson
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This is a comprehensive and frank biography of Stephen Smale, one of the best-known American mathematicians. Smale worked in many areas of mathematics, notably topology and dynamical systems; received the Fields Medal in 1966; and was also famous for his opposition to the Vietnam War. The author, Steve Batterson, is a mathematical grandson of Smale, having received his Ph.D. in 1976 with John Franks, and works in dynamical systems at Emory University. Although the book is aimed at nonmathematicians, it probably will appeal most to mathematicians in fields in which Smale worked and/or to readers with an interest in the politics of the 1950s and 1960s. I found the book fascinating.

The book's opening chapter describes Smale's parents and his childhood through high school. It is the stuff of storybooks: a one-room schoolhouse, an iconoclastic father, a rural (dirt roads) Michigan upbringing, a somewhat lonely child who plays chess but is not a prodigy. If one tries to account for Smale's future success, one is left with good genes and good parenting, for Smale had few of the academic privileges that are thought necessary for success today: no fancy schools with learned teachers and the latest equipment, no immersion into a college-prep high school or summer math camps, no gifted and talented programs, no extracurricular lessons of any sort. Smale was allowed to grow

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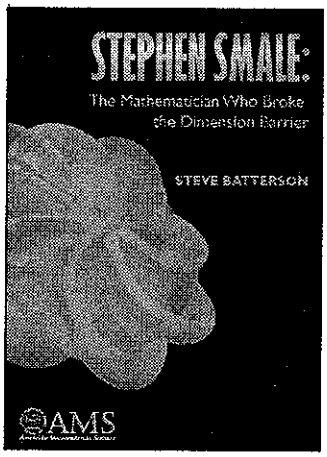

up on his own to a large extent, and he appears to have developed early on his famous independence, as well as a certain willingness to take risks, mathematical and otherwise.

Smale went to the University of Michigan in 1948, receiving a B.S. in 1952 and a Ph.D. in 1956. Batterson writes that these eight years divide into a serious academic freshman year, then four years of political activity and mediocre grades, followed by three years of concentration on mathematics plus marriage.

Smale's political activity included openly joining the Labor Youth League (the Communist Party youth organization) and joining the Communist Party (a fact kept secret for many years). According to Batterson, Smale was quite close to being caught by HUAC (the House Un-American Activities Committee), and being caught might well have resulted in his emigration to Canada.

In view of Smale's independence it is surprising that he joined the Stalinist Communist Party, not known for tolerating independent thinking. His explanation is enlightening:

> ...I was sufficiently skeptical of the country's institutions to the point that I couldn't accept the negative reports about the Soviet Union. I so believed in the goal of a utopian society that brutal means to achieve it could be justified.

> I was unsure of myself on social ground, and the developing social network of leftists around me gave me security. Then, these were the times of McCarthyism, the Rosenberg executions, the Korean War hysteria; the [Communist Party] was the main group giving unqualified resistance to these forces.

Many people today are skeptical of U.S. institutions, but perhaps it is surprising that Smale was not also skeptical of the Soviet Union. But, as I remember the early 1950s, there was more wide-eyed, naive idealism around then than there is today, and Smale's explanation makes sense to me.

It also allows one to understand better the motivation of those on the anti-Communist side. They saw college students as somewhat impressionable young people who could be led astray by the lures of utopian goals and a "social network of leftists". As they believed that Communism was a bad system, it is not so surprising that they used means even as extreme as McCarthyism to combat the leftists. If Smale believed that utopian goals justified brutal means, surely the opposition could believe that their goals justified blacklisting and ruining careers of otherwise decent people. Fortunately we have moved towards the position that it is better to let young radicals learn for themselves rather than to use suppression to the extent done in the 1950s.

Smale became Raoul Bott's first Ph.D. student and wrote an excellent Ph.D. thesis. In the summer of 1956 Smale attended the famous topology conference in Mexico City and met the cream of international topologists. He then went on to an instructorship at the University of Chicago. These were Chicago's glory days and Smale's as well. During his first year he proved his famous theorem that one can turn a 2-sphere in R^3 inside out through immersions.

Smale generalized his immersion theorem to the case of S^n in S^q, $n < q$, and this was followed by a seemingly endless sequence of generalizations: from spheres to manifolds (due to Morris Hirsch), from smooth to the piecewise linear and topological categories, and from immersions to submersions. Eventually came Gromov's nearly ultimate version [1, 2].

Batterson tells us how difficult it was to actually construct a regular homotopy (an arc of immersions) that everted the 2-sphere (Smale gave only a classification of immersions, which implied existence of an eversion). But Batterson does not mention a useful conceptual aid. An immersion into R^3 of the real projective plane, RP^2, and hence also S^2 by the covering map, was well known (e.g., Boy's surface). Furthermore, the normal 0-sphere bundle to RP^2 is an immersed 2-sphere. There is an obvious arc of immersions that switches the two points of each S^0 along the normal line joining them, and this arc of immersions turns the immersed sphere inside out. What remains, and this is still difficult, is to construct an arc of immersions from the standard embedding of the sphere to the immersion of the 0-sphere bundle.

After two years, 1956–58, at Chicago Smale won a National Science Foundation (NSF) fellowship and membership at the Institute for Advanced Study. But he chose to spend spring 1960 at the Istituto de Matemática Pura e Aplicada in Rio de Janeiro with Maurice Peixoto and Elon Lima. This was the setting for famous stories, which we will discuss shortly, of the beaches of Rio, the horseshoe, the Poincaré Conjecture, and later NSF troubles.

Smale had been thinking about the structural stability of dynamical systems after an encounter with Peixoto in 1958. An error in Smale's work, pointed out by Norman Levinson, led Smale to study carefully work of Mary Cartwright and J. L. Littlewood and eventually to discover his "horseshoe", a beautiful and central example of a structurally stable but "chaotic" system.

A few months later, while still thinking on the beaches, Smale saw how to prove the Poincaré Conjecture in dimensions greater than 4. Not long after that he proved the h-cobordism theorem, arguably the most important theorem in topology, at least in the area of the classification of manifolds (the Poincaré conjecture in dimensions greater than 4 is then an easy corollary). There is an excellent survey by Hirsch [3] of Smale's work in differential topology. In mathematical appendices to the book Batterson does a nice job of explaining the horseshoe to the nonexpert; Smale's topological work is discussed only at a much more elementary level.

The early 1960s are covered in Chapters 5–7, and at this point an undercurrent becomes apparent in the book. This undercurrent concerns how Smale is regarded in the mathematical community, whether his recognition is less than he deserves, and explanations of why Smale does not get all the honor he should. Batterson gives various examples, but perhaps the two principal ones concern credit for the Poincaré Conjecture and h-cobordism theorem and whether Smale should have won the Fields Medal in 1962. More generally, Smale's followers in the field of dynamical systems (where many of his students work) have always been very loyal supporters, whereas Batterson suggests that topologists and others have been penurious in their support. This topic may not be worth our time, but it keeps arising in the book.

It was in the *Bulletin of the American Mathematical Society (N.S.)* that Smale announced his proof of the higher-dimensional Poincaré Conjecture, a surprising and striking result. He was still in Brazil and unable to defend his sketch of the proof

in person. John Stallings heard the claim and hints of the method and produced a rather different proof for piecewise linear homotopy spheres. (Much is written on this matter in [3].) I believe that Smale deserves the lion's share of the credit for the higher-dimensional Poincaré Conjecture. That statement is imprecise, and other mathematicians have given their own imprecise statements, attributing a mix of credit to Smale and Stallings. Sometimes they appear to err in Stallings's favor, and sometimes they give too little credit to both.

Batterson wonders if the erroneous attributions occurred because topologists were miffed that Smale left topology in 1961 or because John Milnor's book on the h-cobordism theorem [4] induced topologists to learn the theorem there rather than in Smale's papers. Batterson asks why Smale's name was not attached to either theorem as, he implies, it should have been.

In fact there is no need to hint at any sort of pettiness to explain these things. In general it is safe and wise to be generous with credit; Smale loses much less if too much credit is given to Stallings than Stallings would lose if too little were given to him. And it is widely known that the h-cobordism theorem is Smale's (no conflict there) and the Poincaré Conjecture, being a corollary, is therefore Smale's also. This gets the history reversed, but it is the way the subject is casually taught and learned these days. It is Stallings who gets forgotten or added as an afterthought.

Yes, Smale's name is not attached to either theorem. But things like this happen. The process by which names get attached or do not get attached to particular mathematical results is rather haphazard.

Later Batterson speculates about why Smale did not get a Fields Medals in 1962, when medals went to Milnor and Lars Hörmander. Batterson writes, "Not winning was a tremendous blow to Smale. He attributes his loss, in part, to mathematical politics." Batterson goes on to talk about "influential backers," Smale's following an "unconventional path," and his being deprived of "full credit for the Poincaré Conjecture." This could all be true, but it is beside the point. For one thing, Smale's announcement and proofs were appearing as the Fields Medal committee was deliberating and had not yet withstood the test of time. In addition, the committee knew that Smale could get the Fields Medal four years later, as he did.

To be unable to accept that a negative decision is plausible on purely mathematical grounds (or is sometimes caused by random bumbling) and to suggest that the decision is due to less than admirable motives are occasional and unfortunate characteristics of Batterson's approach to his subject.

The middle third of the book recounts three political issues: Smale's role as organizer of the Vietnam Day protest in Berkeley in May 1965, his press conference in Moscow during the 1966 International Congress of Mathematicians while he was being pursued by HUAC, and NSF unhappiness over Smale's grant. The facts are clearly presented with some of the political background (though there is not much here that cannot be found in [3] and a few other sources), but there is no genuine analysis of the material.

For example, we read that in summer 1966 the NSF was in hot water because a huge geological project called the Mohole had failed. On top of that, Congress was hearing about controversial remarks made by Smale when he was in Moscow as well as when he was "vacationing" in Greece with his family, all while he was receiving summer salary and travel support from NSF. Furthermore, the phrase "doing his best work on the beaches of Rio" surfaced again.

The NSF at first tried to deny Smale some of his summer salary and travel funds but backed down. Then a few months later it tried to split off Smale from a large grant of which he was principal investigator, claiming he was an unsuitable administrator for a large grant. The NSF eventually backed off again.

Smale had much the best of the arguments. It was irrelevant how he spent his time: surfing, vacationing, sleeping. He produced. The right argument for NSF, then or today, is that the best predictor of future success at research is past success. If the last three or more years have produced good enough work, then NSF should award another three-year grant. Why pay any attention to anything else, including even the current proposal, unless it is nonsense? I think this philosophy is what carried the day for Smale, but it is rarely put quite this bluntly. Here is why.

In the experimental sciences, which get most of NSF's money, a principal investigator proposes to carry out experiments. These entail long hours in a laboratory, and the value and probability of success of a proposed experiment are not so hard to assess. No long hours on a beach. No impossibility of knowing whether an attack on a hard problem is going to succeed or something completely surprising will turn up. If an experimentalist is vacationing, NSF has reason to worry. It is not easy to crack down on the experimentalists while shrugging off mathematicians lolling on the beach. When considering characteristics of various disciplines, mathematics is usually an outlier, but one can push our "differentness" only so far.

Furthermore, one can worry about a discipline in which it becomes fashionable to boast of great results without hard, disciplined work in the background. One can imagine a whole discipline developing casual habits with declining productivity. If, as many parents do, we worry about getting our children into the right schools with the right friends and environment in the hopes that good things will rub off on our children, then should we be surprised

that the NSF and Congress were uncomfortable with the stories about Smale? Of course, the NSF should not have hassled Smale as it did, but Smale could also have chosen to be less confrontational.

Yet Smale was being confrontational over a very serious matter, the Vietnam War—a "horrible war," as he called it. The book describes in detail Smale's role in helping to organize the 33-hour "teach-in" called Vietnam Day and later his role in trying to stop troop trains as they went through Berkeley. Another chapter describes Smale's press conference on the steps of Moscow University and the ridiculous media coverage of HUAC's attempt to subpoena Smale. These are good stories, and again Smale had much the best of the arguments, with one proviso.

In the mid-1960s the U.S. was split over the conduct of the war, but even then and certainly with hindsight one could make practical objections to the war. One could believe that it was correct (perhaps necessary, perhaps even honorable) to spend some effort towards keeping the Communists from taking over South Vietnam (some thought this was just a civil war, with North Vietnam trying to unite their country after French colonialism failed). But as a practical matter one could argue that by 1965 intervention in Vietnam was not working and that the cost of "saving" South Vietnam was already too large and was growing larger, with no "light at the end of the tunnel".

However, a significant fraction of the anti-war activists went further, essentially taking the side of the North Vietnamese rather than just opposing the U.S. involvement as unworkable. Batterson does not say much about Smale's views, but Smale appears to belong to this fraction of activists. Again, as with Smale's membership in the Communist Party in college, one wonders why he was not more skeptical about the North Vietnamese.

In addition to discussing Smale's mathematical and political activities, the book covers other important parts of his life. Chapter 9 describes how Smale became a world-class collector of minerals, particularly gemstones. His collection apparently ranks among the top five of private collectors. Furthermore, he became an avid and very proficient photographer of his minerals.

Chapter 10 contains an interesting account of Smale's sailing trip from Berkeley to the Marquesas (3,000 miles) to Hawaii and back with crew members Charles Pugh and Welington de Melo. Chapter 11, titled "Other People", discusses Smale's family and his students and gives a rather frank account of Smale's role in the Jenny Harrison tenure case at Berkeley.

Chapter 12 provides a laudatory assessment of "Smale, the Mathematician". Throughout the book Batterson repeatedly uses the word "audacity" in connection with Smale. *Webster's Dictionary* defines audacity as "a: intrepid boldness; b: bold or arrogant disregard of normal restraints." It is not always clear which definition best fits Smale.

A continuing theme in the book is a comparison of Smale with other competitors in whatever endeavor is being considered. Many readers would find the constant comparing to be considerably overdone in the book, but perhaps it is appropriate in a biography of Smale, a serious competitor. It also provokes this reviewer to a bit of assessment based on what the book says.

Smale deserved his Fields Medal for his work in topology and his early work (e.g., the horseshoe) in dynamical systems. Some have questions about the value of his "applied" work that came after his work in dynamical systems, but he should be seen as an excellent example of how to avoid stagnation in a field one has pioneered and instead to stay active by trying new subjects.

The nonmathematical side of his public life involves, most of all, leftist politics. He seems to have been an activist rather than a thinker, as far as I can tell. I would have preferred that he devote a fraction of his time as an activist to writing down careful arguments for his beliefs. One can try to rally the faithful, or to tip the fence-sitters, or to weaken the resolve of one's opponents. Usually activists throw red meat to the faithful, but I respect the much harder task of making a more universal argument. Apparently this was not Smale's forte.

All in all, Steve Smale is a first-rate Fields Medalist who has led a rich and varied life.

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