The constructions in "Stable Homeomorphisms" can be extended to investigate homeomorphisms of $B^k \times R^n$ which fix $\partial B^k \times R^n$. Since $B^k \times R^n = B^k \times \text{int } B^n$, we get a tool for isotoping a topological handle $B^k \times B^n$ to a PL handle modulo $\partial B^k \times B^n$. Then the theorems of "Stable Homeomorphisms" generalize to manifolds.

Theorem: Let $M^m$ be a topological handlebody, closed, open, or with boundary. Then the space of homeomorphisms of $M^m$, with the compact-open topology, is locally contractible.

Conjecture 1 (k,n): Let $h : B^k \times T^n \longrightarrow W^{k+n}$ be a homeomorphism which is PL on $\partial B^k \times T^n$. Then $h$ is homotopic to a PL homeomorphism $g$ where $g = h$ on $\partial B^k \times T^n$.

Conjecture 2 (k,n): Let $f : B^k \times T^n \longrightarrow B^k \times T^n$ be a PL homeomorphism sufficiently close to the identity, with $f = \text{id.}$ on $\partial B^k \times T^n$. Then $f$ is PL isotopic to the identity modulo $\partial B^k \times T^n$.

Theorem: Let $Q^q$ be a PL manifold and $P^q$ a non-empty open submanifold with $\partial Q \subset P$. Let $h$ be a homeomorphism of $Q$ which is PL on $P$. Then if $q \geq 6$ and either conjecture holds for all $k,n$ with $\overset{\geq 2}{k} + n = q$, then $h$ is $\varepsilon$-isotopic to a PL homeomorphism modulo a slightly smaller manifold $P'$.

Corollary: If $q \geq 6$ and either conjecture holds for all $\overset{\geq 1}{k},n$ with $k + n = q$, then any stable manifold is triangulable. Then simply connected manifolds are triangulable since they are stable.