Math 128a Midterm 1

1. Round

$$e = 2.71828182845905\cdots$$

to 5 decimal digits. Evaluate the absolute and relative errors and the number of significant digits. Repeat this for $e \times 10^{10^3}$.

2. Consider the following iteration:

$$x_{n+1} = x_n - \sin(ax_n)\,, \quad x_0 = 1/2$$

Describe the convergence properties of this sequence (including the rate, if convergent) in two cases:

$$\text{a) } a = \pi \ , \qquad\qquad \text{b) } a = 1 \ .$$

3. Construct the Hermite polynomial, $H(x)$, which interpolates

$$\sin\left(\frac{\pi x}{2}\right) \quad \text{at } x_0 = 0\,,\ x_1 = 1\,.$$

Show that

$$|\sin(\pi x/2) - H(x)| \le \frac{1}{24}\,, \quad 0 \le x \le 1\,.$$