Mathematics 1B.

Practice Midterm 1

Problem 1. Evaluate the following (indefinite) integrals

a)
$$\int (x^{1/3} + x^{1/4})^{-1} dx$$

b) $\int \frac{x}{x^3 + 1} dx$

Problem 2. Evaluate the following (definite) integrals

a)
$$\int_0^1 x^2 \tan^{-1} x dx$$

b) $\int_0^\infty e^{-x} (1 - e^{-2x})^{\frac{1}{2}} dx$

b)
$$\int_0^{\infty} e^{-x} (1-e^{-x})^2 dx$$

Problem 3. a) Suppose that f(x) is a function defined on [a, b]. State the formula for the area of the surface of revolution obtained by rotating the graph of f around the x axis.

b) Find that area in the case when $f(x) = 1 + \sqrt{1 - x^2}$ and a = 0, b = 1.

Problem 4. Determine (providing an explanation) the convergence or divergence of the following series.

a)
$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n}$$

b)
$$\sum_{n=2}^{\infty} \frac{n \ln n}{n^3 + 1}$$

c)
$$\sum_{n=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{1 \cdot 4 \cdot 7 \cdots (3n+1)}$$

Problem 5. Estimate the error in approximating the following series by the sum of its first 100 terms:

a)
$$\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$$
b)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$