Mathematics 1B. **Fall Semester 2006**

Sample Final Exam

1.(20 points) Evaluate the following integrals:

$$a)\quad \int (1+\sqrt{1+x})^{-1}dx$$

$$b)\quad \int x\ln\sqrt{1+x^2}dx$$

$$c)\quad \int_0^{\pi/6} \tan^4 x\sec^2 x dx$$

$$d)\quad \int \frac{x}{(x^2+x+1)(x^2+1)}dx$$

2. (15 points) Find the convergence radius for the series:

$$a)\quad \sum_{n=1}^{\infty}(1+n)^2x^n$$

$$b)\quad \sum_{n=0}^{\infty} 4^n x^{n^2}$$

$$c)\quad \sum_{n=2}^{\infty}(-1)^n \sin\frac{1}{n}x^{2n-3}$$

3. (15 points) Solve the second order differential equations:

$$a)\quad y''-y'=e^x, \qquad y(0)=y'(0)=0$$

$$b)\quad y''+y=\sec x, \qquad y(0)=0,\ y'(0)=1$$

$$c)\quad y''-2y+2=0, \qquad y(0)=1,\ y'(0)=0$$

4. (10 points) Solve the first order differential equations:

$$a)\quad y'+xy=x,\quad y(0)=\frac{1}{2}$$

$$b)\quad y'=(1+x)/(xy),\ x>0,\ y(1)=2$$

5. (5 points) The series

$$\sum_{n=0}^{\infty} 2^n \sin\left(\frac{1}{3^n}\right)$$

a) diverges b) converges by the alternating series test c) converges by the root test d) converges by the comparison test e) converges by the integral test.

6. (5points)

$$\int_0^x e^{t^2}\,dx = \qquad a) e^{x^2}-1 \qquad b) \sum_{n=0}^{\infty} x^{2n} n!$$

$$c) \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)n!} \qquad d) \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(n+1)!} \qquad e) \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)(2n)!}$$

7. (5 points) A body with mass 2 is attached to a spring with elastic constant 3 and friction coefficient 4. The equation of motion for the string is

$$a)\ 2x'' + 3x' - 4x = 0 \qquad b)\ 3x'' - 4x' + 2x = 0 \qquad c)\ 6x'' + 4x = 0$$

$$d)\ 2x'' + 4x' + 3x = 0 \qquad e)\ 4x'' + 3x' + 2x = 0$$

8. (5 points) Choose the optimal estimate among the ones given:

$$\sum_{n=101}^{\infty} n^{-\frac{3}{2}} \le \qquad a)\ 0.001 \quad b)\ 0.01 \quad c)\ \ 0.0033 \quad d)\ 0.1 \quad e)\ \ 0.33$$

9.(5 points) The series

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{2p}}, \qquad p > 0$$

a) converges conditionally for all $p > 0$.
b) converges absolutely for all $p > 0$.
c) converges for $p > 1/2$ and diverges for $p \le 1/2$.
d) converges absolutely for $p > 1/2$ and converges conditionally for $p \le 1/2$.
e)converges absolutely for $p > 1$ and converges conditionally for $p \le 1$.

10. (5 points) The initial value problem

$$y'' + x^2 y' = 0, \qquad y(0) = 0,\ y'(0) = 1/3$$

is solved by the following power series:

$$a)\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(3n+1)3^n n!} \qquad b)\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(3n+1)3^{n+1} n!}$$

$$c)\ \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{3n+1}}{(3n+1)3^n n!} \qquad d)\ \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{3n!}$$

e) none of the above.

11. (5 points) The function $f(x) = \ln(1 + 2x + x^2)$ is best approximated near $x = 0$ by a) 0 b) $2x - x^2$ c) $2x$ d) $2x + 2x^2$ e) $2x + x^3$

12. (5 points) The improper integral

$$\int_0^\infty \frac{x+1}{x^{\frac{3}{2}}} dx$$

is

a) convergent
b) convergent at 0, divergent at ∞.
c) convergent at ∞, divergent at 0.
d) divergent at both 0 or ∞.
e) none of the above.