Some quizzes

April 3, 2013

Suppose you are putting 200 pigeons into some holes, and you want to guarantee that there is at least one hole with at least 4 pigeons in it no matter how you put the pigeons into the holes. What is the largest number of holes that works?

Give a combinatorial proof of $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

How many ways are there to distribute 3 (indistinguishable) oranges, 2 (indistinguishable) apples, and 3 (indistinguishable) bananas among 8 (distinguishable) people? Express your final answer in base ten.

Find the inverse of 12 modulo 257.

Solve $12x \equiv 13 \pmod{257}$.

Prove that $1 + 3 + \cdots + (2n - 1) = n^2$ for $n \geq 1$.

Prove using strong induction that any amount of postage greater than or equal to 12 cents can be made using only 3 and 7 cent stamps.

Let a and b be positive integers. Assume that a is a 2-digit number in base 10. Is it possible that it takes ten divisions when using the Euclidean algorithm to calculate $\gcd(a, b)$? (Either prove it is not possible, or give an example showing its possible.)