Quiz questions given through February 15, 2013

a) Make a truth table for $p \to q$ and $q \to p$. b) State whether these two formulas are logically equivalent or not and circle the relevant portions of the truth table for justification.

Express $\exists x P(x)$ in terms of \forall .

Prove that $\sqrt{2}$ is irrational.

a) Make a truth table for $p \to q$ and $\neg q \to \neg p$. b) State whether these two formulas are logically equivalent or not and circle the relevant portions of the truth table for justification.

Express $\forall x (P(x) \land Q(x))$ in terms of \exists and \lor .

Prove that the product of two odd numbers is always odd.

a) Make a truth table for $p \to q$ and $\neg p \to \neg q$. b) State whether these two formulas are logically equivalent or not and circle the relevant portions of the truth table for justification.

Express $\neg \exists x \neg P(x)$ in terms of \forall .

Let x and y be integers. Prove that if xy and x + y are even, then x and y are even too.

Use truth tables to prove that $\neg(p \lor q)$ is equivalent to $\neg p \land \neg q$.

Given sets A and B, prove that the complement of the intersection of A and B is equal to the union of the complement of A and the complement of B.

Determine whether

a)
$$\forall x [\exists y (x \leq y)]$$
 b) $\exists y [\forall x (x \leq y)]$

are true or false when the domain is \mathbb{Z} . Explain.

Is the compound proposition $(p \to q) \land (q \to r)$ logically *equivalent* to $p \to r$? Hint: It is not. Explain.

Prove that $\sqrt{2} + \sqrt{3}$ is irrational assuming you know that $\sqrt{2}$ is irrational. Determine whether

a)
$$\forall x [\exists y(xy=1)]$$
 b) $\exists y [\forall x(xy=1)]$

are true or false when the domain is $\mathbb{Q} - \{0\}$. Explain.

Is the compound proposition $(p \to q) \land (q \to r)$ logically *equivalent* to $p \to r$? Hint: It is not. Explain.

Prove that $\sqrt{2} - \sqrt{5}$ is irrational assuming you know that $\sqrt{2}$ is irrational.