Math 250B

Homework due May 16, 2003

Recall that $\mathbf{Z}_p$ denotes the p-adic completion of $\mathbf{Z}$ and that $\hat{\mathbf{Z}}$ denotes the profinite completion of $\mathbf{Z}$.

1. Show that -1 is a square in $\mathbf{Z}_5$.

2. Let E be the set consisting of all non-negative integers, together with an extra element ∞. A *supernatural number* is a formal product $\prod_p p^{e_p}$, where the product runs over all primes p and where the exponents e_p are elements of E. If m is a supernatural number, let $m\hat{\mathbf{Z}}$ be the intersection of the groups $n\hat{\mathbf{Z}}$, taken over all positive integers n that divide m. Show that the set of closed subgroups of $\hat{\mathbf{Z}}$ corresponds bijectively with the set of supernatural numbers under the map $m \mapsto m\hat{\mathbf{Z}}$.

3. Let K and L be extensions of k inside a large field Ω (as in Chapter VIII, §3 of the textbook). Is it true that K and L are linearly disjoint over k if and only if the natural map $K \otimes_k L \to \Omega$ is injective?

4. At the beginning of the proof of Theorem 4.13 on page 367, our author says "From the hypotheses, we deduce that K is free from the algebraic closure L^a of L over k." How do we deduce this?

5. Do problems 2–7 at the end of Chapter VII (pp. 374–375).