Mathematics 113 — Professor K. A. Ribet

Last Midterm Exam — October 31, 2013

Afternoon Edition

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly in *complete sentences*. Your explanations are your only representative when your work is being graded.

The problems have equal weight. We write $|G|$ for the order of a group G.

1. Find the number of conjugates of $(1\,2\,3)(4\,5\,6)$ in A_6. (For this problem, and the ones below, be sure to explain your work in complete English sentences.)

2. Let p be an odd prime, and let G be a dihedral group D_{2n}. Show that all p-Sylow subgroups of G are cyclic. Find the number of such subgroups.

3. Suppose that G is a finite group and that H is a subgroup of G. Let $N = N_G(H)$ be the normalizer of H.

a. Let $H_1 = H, H_2, H_3, \ldots, H_k$ be the distinct conjugates of H in G. Prove the formula

$$\sum_{i=1}^{k} |H_i| = |H| \cdot (G : N) = |G|/(N : H).$$

b. If $H \neq G$, show that $\bigcup_{i=1}^{k} H_i \neq G$.

4. Let G be a group (possibly infinite) and let H be a subgroup of G for which the set G/H is finite. Use the action of G by left multiplication on G/H to show that there is a normal subgroup N of G such that $N \subseteq H$ and such that G/N is a finite group.

5. Let G be a group.

a. For each $g \in G$, let σ_g be the inner automorphism "conjugation by g." Suppose that φ is an automorphism of G. Establish the formula $\varphi\sigma_g\varphi^{-1} = \sigma_{\varphi(g)}$.

b. If G has trivial center and φ commutes with all σ_g, show that φ is the identity map.