Mathematics 113 | Professor K. A. Ribet
First Midterm Exam | September 26, 2013

Morning Edition

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly in *complete sentences.* Your explanations are your only representative when your work is being graded.

The problems have equal weight.

1. If H is a subgroup of $\mathbf{Z}$ (the group of integers under addition), prove that there is an integer $n \geq 0$ such that H is the set of integer multiples of n.

2. Let n and m be integers ≥ 3. Suppose that there is a surjective ("onto") homomorphism $D_{2n} \to D_{2m}$. Show that n is a multiple of m.

3. If n is a positive integer, recall that $\mathbf{Z}/n\mathbf{Z}$ is the group of mod n integers under addition and that $(\mathbf{Z}/n\mathbf{Z})^*$ is the group of invertible mod n integers under multiplication. Recall further that a mod n is invertible if and only if a and n are relatively prime (i.e., have gcd $= 1$). The group $G = (\mathbf{Z}/n\mathbf{Z})^*$ operates on the set $A = \mathbf{Z}/n\mathbf{Z}$ by multiplication: for $g \in G$ and $a \in A$, $g \cdot a$ is the product of g and a mod n.

For $a \in \mathbf{Z}/n\mathbf{Z}$, establish a formula (in terms of a and n) for the order of the stabilizer G_a.

4. Find the 20th power of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 4 & 8 & 1 & 10 & 5 & 6 & 9 & 3 & 2 \end{pmatrix}$.

5. Let Ω be the set of positive integers. For each positive integer n, describe an element of order n in the symmetric group S_Ω.