Math 113H

Professor K. A. Ribet
Midterm Exam April 5, 1991

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1. (6 points) Show that all groups of order 22 are either cyclic or dihedral.

2. (7 points) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 7 & 6 & 8 & 1 & 5 & 2 \end{pmatrix}$.

a. Write σ as a product of disjoint cycles.

b. Find $\operatorname{sgn} \sigma$.

c. Calculate $\tau\sigma\tau^{-1}$, where $\tau = (1\,2\,3)(4\,5\,6)$.

3. (6 points)

a. For which n does the symmetric group S_n have an element of order 15?

b. For which n does S_n have a subgroup of order 15?

4. (7 points) Let N be a normal subgroup of the group G. Assume that $N \neq (e)$ and that G is finite with p-power order (where p is a prime). Show that $N \cap Z \neq (e)$, where Z is the center of G.