Math H113 Fall, 1984

First Midterm Exam
Professor K. Ribet

These questions were given on an 50-minute exam in 1984. The textbook was probably Herstein's "Topics in Algebra."

1. Suppose that G is a finite group.

a. Define what is meant by the order of an element g of G and by the order of a subgroup H of G. Given an element g of G, is it always possible to find a subgroup H of G whose order is equal to that of g?

b. Show that G has an even number of elements if and only if the set

$$S = \{ g \in G \mid g^{-1} = g \}$$

has more than one element.

c. Show that G is abelian if $S = G$.

d. Show that S is a subgroup of G if G is abelian.

e. Show, by example, that S may fail to be a subgroup if G is not abelian.

2. Use the method indicated in class to find integers x and y such that $37x + 10y = 1$.