We're having fun in Math 10A?

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August 31, 2017

Yes

We are having fun in Math 10A.

But please put away all phones, tables, laptops,. . . . Thanks!!

Feedback from a student

- Slides are awesome! More slides!
- Chalk boards are way less awesome...
- Do not write in yellow chalk it's incredibly hard to see.
- Also write bigger please!

Do you agree?

When I'm writing too small, let me know.

Social events

Yesterday's breakfast:

There are still four slots for the breakfast at 8AM on Wednesday, September 20.

The next pop-in lunch will be tomorrow at 11:45AM at the Faculty Club.

More sequences

You've probably heard of the Fibonacci numbers

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots,$$

which appear in Math 10B. The rule is that each number is the sum of the preceding two.

That's one sequence: the Fibonacci numbers.

A second is the sequence of *ratios* of successive Fibonacci numbers:

$$1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13.$$

Numerically:

$$1, 2, 1.5, 1.66, 1.6, 1.625, 1.615384.$$

These ratios go alternately up and down, as you can see from the examples. (2 is bigger than 1, 1.5 is less than 2, 1.66 is bigger than 1.5, etc.)

Fact: The ratios approach $\frac{1+\sqrt{5}}{2} \approx 1.6180$.

This statement can be deduced pretty easily from an explicit formula for the *n*th Fibonacci number that you're likely to see in Math 10B.

Fake news

The number of kilometers in a mile is equal to $\dfrac{1+\sqrt{5}}{2}$.

Actually, the number of kilometers in a mile is roughly 1.60934, as compared to 1.6180, the numerical value of $\dfrac{1+\sqrt{5}}{2}$.

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Powers

What are

$$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n, \quad 2^n, \quad (-1)^n, \quad \left(\frac{-1}{3}\right)^n?$$

Answers

About $\lim_{n\to\infty} a^n$:

- it's 0 for $-1 < a < 1$;
- it's 1 for $a = 1$ (constant sequence)
- it doesn't exist for $a = -1$ (oscillates back and forth)
- it doesn't exist for $a > 1$ (goes to infinity) or for $a < -1$ (goes to infinity in absolute value but is alternately positive and negative).

Geometric series

Let a be a real number, and let

$$s_n = 1 + a + a^2 + \cdots + a^n \text{ for } n \geq 0.$$

What can we say about $\lim_{n\to\infty} s_n$?

The infinite series

$$1 + a + a^2 + \cdots\cdots$$

is a suave way of writing the limit.

What is the infinite sum $1 + a + a^2 + \cdots\cdots$?

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Special case: $a = 1$. Then $s_n = n + 1$, so $s_n \to \infty$. With this understood, we can and will assume $a \neq 1$.

Magic formula:

$$s_n = \frac{a^{n+1} - 1}{a - 1} \text{ (when } a \neq 1).$$

The proof is to multiply out $s_n(a - 1)$. Since s_n has $n + 1$ terms, the produt has $2(n + 1)$ terms, half with a $+$ sign and half with a $-$ sign. The terms cancel out in pairs, except for $+a^{n+1}$ and -1, which remain.

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Infinite geometric series

Conclusion:

$$1 + a + a^2 + \cdots = \lim_{n\to\infty} \frac{1 - a^{n+1}}{1 - a} = \frac{1}{1-a} \text{ if } -1 < a < +1.$$

The proof is that $a^{n+1} \to 0$ when $-1 < a < +1$.

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Some fractions

Find

$$\lim_{n\to\infty} \frac{n^5 - 3n^4 + n^2 - 1}{n^6 - 5n^3 + 18n - 3}.$$

This is 0 because "n^6 is a lot bigger than n^5"—but there's a more satisfactory explanation that you see by dividing numerator and denominator by n^6. (I'll do at board.)

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As $n \to \infty$, the fraction approaches $+\infty$ (same analysis as on previous slide). We might say:

- the limit doesn't exist (meaning that there's no number that serves as limit);
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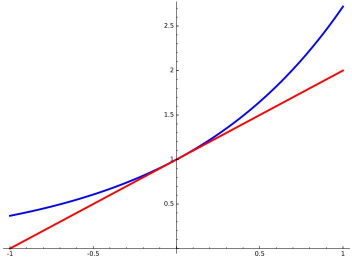
What is a derivative?

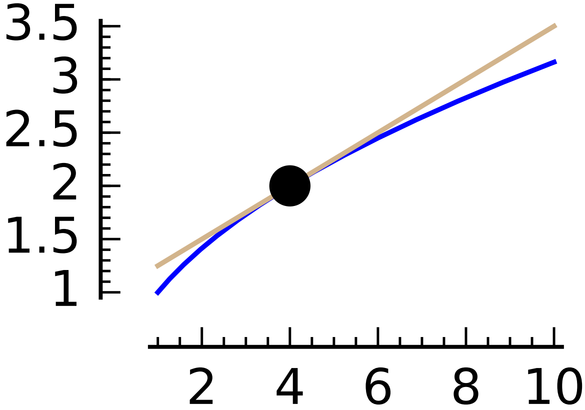
We want to define the derivative of $f(x)$ at $x = a$ when f is a function (on some interval, say) and a is a point inside the domain of defintion of f.

The derivative of f at a, denoted $f'(a)$ is the slope of the line that kisses the curve $y = f(x)$ at the point $(a, f(a))$.



The derivative of e^x at $x = 0$ is 1.

Another example



What is the derivative of $y = \sqrt{x}$ at $x = 4$?

A new bookseller

Josh Adams